Is Mereology Ontologically Innocent? Well, it Depends . . . [*]

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Abstract

[395] Mereology, the theory of parts and wholes, is sometimes used as a framework for categorisation because it is regarded as ontologically innocent in the sense that the mereological fusion of some entities is nothing over and above the entities. In this paper it is argued that an adequate answer to the question of whether the thesis of the ontological innocence of mereology holds relies crucially on the underlying theory of reference. It is then shown that upholding the thesis comes at high costs, viz. at the cost of a quite strong logical background theory or at paradoxical ways of predicating and counting.

Keywords: *unrestricted composition*, *composition as identity*, *plural quantification*, *plural predication*

1 Introduction

Categorisation is a fundamental operation and is based only on more fundamental theories of abstraction: formally speaking categories may be considered to be sets, classes, collections, fusions, aggregations, etc. But if categorisation is based on such an abstraction one may wonder whether such a

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basis is ontologically harmless—fine shaved as Ockham—or ontologically quite heavily laden—like Plato's beard. Mereology, the theory of parts and wholes, is often taken to be an adequate framework for [396] categorisation since it is less heavily laden than set theory. But some metaphysicians and ontologists think that it is still too heavy (cf., e.g., Bohn 2009, p.27). In particular the so-called *principle of unrestricted fusion or composition* is under attack, since it allows the fusion or composition of any entities. The critique is, that "unrestricted [fusion or composition] appears to go too far, for [... it] also commits the theory to the existence of a large variety of *prima facie* implausible, unheard-of mereological composites—a large variety of 'junk' in the good old sense of the word" (cf. Varzi 2014, sect.4.5). It does not make one wonder that from such a point of view demands of restricting the fusion or composition principle to operations with outcomes that are in a natural way categorisable evolved (cf., e.g., Bohn 2009).

But there is another way out of the problem of "junk" entities through mereological abstraction: One might claim that fusions and compositions are nothing over and above the entities the abstraction operation is performed on. According to such a thesis on the ontological innocence the fusion or composition of some entities x and y is identical with x and y; and since identity is a logical relation, the thesis whereby mereology is ontologically innocent is just the claim that "generating" entities with the help of the mereological fusion or composition operation is a special kind of doing logical abstraction.

In a recent collection the debate about the thesis of composition *as* identity was relaunched (Cotnoir and Baxter 2014). Whereas, e.g. (Sider 2014) investigates consequences of this thesis for the framework of plural quantification, this paper investigates the question of whether this approach to the problem is plausible in the framework of plural predication. We will show that an adequate answer to the question of whether the thesis of ontological innocence of mereology holds relies crucially on the underlying theory of reference. Given a singular theory of reference—as, e.g., provided in standard formal semantics—one can easily show that the thesis fails, but if one assumes a plural theory of reference—as, e.g., introduced in 1929 by Stanisław Leśniewski, the founder of mereology—there is a way to make some sense of the thesis. Nevertheless we will see that upholding the innocence thesis actually comes at high costs: (i) accepting full plural quantification theory as *logic* or (ii) accepting some kind of *gavagaian* theory of reference or (iii) accepting paradoxical ways of counting.

In our argumentation we will first describe two common approaches to ontological innocence (section 2), namely the identification approach, claiming that a whole is identical with its parts, and the counting approach, claiming that a whole is the many parts counted as one thing. Afterwards we will discuss an answer to this question and problems of it in the framework of plural quantification theory (section 3). Subsequently we will discuss it in the framework of plural predication by introducing four different theories of predication (section 4) and showing that stating the innocence thesis comes at high costs both, within the identification (section 5) and the counting approach (section 6). We will conclude by providing a short summary of the results and the consequences one may draw from them (section 7).

Note that under 'mereology' we understand here only the extensional theory of the parthood-relation in contradistinction to an intensional one as, e.g., provided in (Simons 1987). The minimal requirement we assume for the parthood relation is that it is a partial order and that a fusion or composition operation is well defined on it, i.e. [397] the existence and uniqueness requirements of such a definition are satisfied. Regarding the operation we will restrict our investigation to the finite fusion or composition of entities, expressible with a binary operation *sum*. This restriction is not essential to our argument on the ontological blameworthyness of mereology since our result for the finite case also transmits to the infinite one (if finite fusion or composition is already ontologically blameworthy, then of course also finite or infinite).

2 Two Theses of Ontological Innocence

Summing up the discussion of ontological innocence of mereology at least partly, one can distinguish two main approaches: sometimes authors stress more the problem of identifying composed entities and sometimes they emphasise more the problem of counting them. Paradigmatic for the first approach is (Lewis 1991). An excellent example for the second approach is (Baxter 1988b). Of course both approaches are closely connected since counting depends on identification. Classical logic allows us to express this fact easily: We can say, e.g., that there are exactly two entities, if there are non-identical entities x and y and if every entity of the universe of discourse is identical with x or y;

Nevertheless we are going to hold the identification and the counting approach separated, because this separation allows us a more sophisticated and illuminating discussion of theories on the innocence thesis.

The first approach to the thesis of ontological innocence of mereology is not primarily concerned with counting individuals, but directly with identifying them, especially with identifying compositions. Puzzling claims are of the kind:

> "The fusion of the *xs* just *is* the *xs*." (Lewis 1991, p.81)
>
> "The *xs* just *are* the fusion of the *xs*." (Lewis 1991, p.81)
>
> "The 'are' of composition is, so to speak, the plural form of the 'is' of identity." (Lewis 1991, p.82)
>
> "Mereological wholes are identical with all their parts taken together." (cf. Armstrong 1997, p.12)

Defenders of the thesis of ontological innocence of mereology within the identification approach claim that the composition of two entities is identical with the entities (cf. the citations above). Opponents of this thesis deny this claim and argue as follows (cf. Inwagen 1994; and similar Yi 1999, p.142, p.146):

1. Assume $x \neq y$ and let z be the composition of x and y: $z = sum(x, y)$.
2. According to defenders of the innocence thesis it holds that the composition is identical with its parts, so assume $z = x$ and $z = y$.
3. The relation of being an improper part ($\preceq$) is reflexive, so it holds $z \preceq z$.
4. As far as x is different from y (1.), the composition of x and y, that is z, is neither an improper part of x nor of y: $z \not\preceq x$ and $z \not\preceq y$.
5. But by the principle of *indiscernibility of identicals* it follows from 2. and 3. that $z \preceq x$ and $z \preceq y$.

[398] Inasmuch as the derivations of 3. and 4. are grounded on basic mereological facts and inasmuch as one accepts the mereological composition principle supposed in 1., one has to blame assumption 2. for the contradiction of 4. and 5.. Opponents of the innocence thesis ascribe, as indicated in 2., this assumption to defenders of the thesis. But this ascription seems to be not very benevolent: it is interpreting a disputant in a not very interesting way, namely as claiming something which contradicts basic mereological facts.

One may ask for a more benevolent interpretation of the claim of the defenders of the innocence thesis: According to a more benevolent interpretation one does not take the second occurrence of 'and' in 'The composition of x and y is identical with x and y.' to be primary, but the 'identical with'. So, instead of reconstructing this claim as '$sum(x, y) = x$ & $sum(x, y) = y$', it seems to be more benevolent and by this correct to reconstruct it in some way like '$sum(x, y) = (x$ & $y)$'. But what does it mean to build up a single term with the help of '&'—we are used to find 'and' between sentences or elliptic formulations within sentences like 'Tris and Iseult are human beings.'? Well, this is not the whole story! Take as example the sentence 'Tris and Iseult are in love.'. One can rephrase it as 'Tris is in love and Iseult is in love.' or as 'Tris is in love with Iseult.'. In the first case the 'and', in the second case the 'are in love' is taken to be primary. Instead of the unary predicate for being in love we use in the second case a more complex one, the binary predicate for being in love with someone.

We have given an analogy now, but it is not fully established because we need to know how to construct an analogue to the rephrase of the second case for our compositional claim. What is the more complex form of the binary predicate for being identical? It seems to make no sense to introduce analogously to the 'is in love' and 'is in love with' case a 3-ary predicate for identity, because the only sense we can make of such an expression is the—by definitions redundant—concept of being identical with something that is identical with something. But what else gives us a more sophisticated structure? The answer seems to be straightforward, if one takes the work of the main defender of the innocence thesis, David Lewis, seriously: It is to do plural referring (cf. Lewis 1991, pp.62ff). So with the term right to the identity sign in our reconstruction of the compositional claim we refer not only to x and also not only to y, but to *x and y*.

There are different ways to implement plural referring into formal systems. One way is to allow quantification with so-called plural variables, as Lewis, following George Boolos (cf. Boolos 1984), suggests. Another way, which is more in the tradition of the founder of mereology, Leśniewski, is to do plural predication. In this paper we will just shortly discuss the former option in the next section and then follow up the line of the later one; this strategy bears the features that our discussion can be easily embedded into classical logic and it allows us also to make easily comparisons of different mereologies.

Before we come to the formal investigation, we give just some intuitions to plural predication that allow us to state the innocence thesis within the identification approach clearly. Classically seen, predication, i.e. singular predication, with the help of the 'is' in 'Iseult is beautiful.' is just the action of stating that the entity referred to by 'Iseult' is identical with one of the entities that are beautiful. Contrary to this, one [399] way of doing plural predication is to state that each entity referred to by 'Iseult' is identical with one of the entities that are beautiful. Since 'Iseult' is usually used for referring to exactly one entity, doing classical predication or doing plural predication makes no difference here. But things disperse if we consider a sentence similar to our 'and'-case above: 'Tris and Iseult are beautiful.' is classically to be understood as: the entity referred to by 'Tris' is one of the entities that are beautiful and the entity referred to by 'Iseult' is one of the entities that are beautiful. In doing plural predication it can be interpreted as: each entity referred to by 'Tris and Iseult', that are Tris and Iseult, is identical with one of the beautiful entities. Later on we will introduce different theories of predication with the help of axioms for the sign 'ε' (which was used by Leśniewski in order to formalise the polish predication-particle 'jest'). But for the moment it is enough to read 'ε' as representation of the 'is' of predication in the sense we just described. Following this intuition, we can formalise the claim that an expression 'z_1' refers exactly to the entities x and y by claiming that x is a z_1 and y is a z_1 and that all entities that are z_1, are x or y. So our auxiliary expression above (that is '$(x \,\&\, y)$') can be replaced by 'z_1', if we assume that z_1 is the z for which it holds: $x \varepsilon z \,\&\, y \varepsilon z \,\&\, \forall z_2(z_2 \varepsilon z \rightarrow z_2 \varepsilon x \vee z_2 \varepsilon y)$. With the help of this pre-formalism we are able to state the innocence thesis within the identification approach more clearly:

OII A mereology M is ontologically innocent in the identification approach iff $M \cup \{\forall x \forall y \forall z(sum(x,y) = z \leftrightarrow (x \varepsilon z \,\&\, y \varepsilon z \,\&\, \forall z_1(z_1 \varepsilon z \rightarrow z_1 \varepsilon x \vee z_1 \varepsilon y)))\}$ is consistent.
Thesis: Mereology is ontologically innocent in this sense.

In a nutshell: defenders and opponents of the innocence thesis within the identification approach state that the composition operation of a mereology M is ontologically innocent, if M is compatible with the claim that the composition of some entities x and y is identical with x and y (plurally referred to) and that is to say that mereology is ontologically innocent, if it makes any sense to claim that the composition operation of mereology (*sum*) is a harmless way of referring plurally.

Let us now come to a characterisation of the innocence thesis within the counting approach! Here one tries to solve puzzling claims of the kind:

"The whole is the many parts counted as one thing." (cf. Baxter 1988a, p.578; and Lewis 1991, p.83)

"If you draw up an inventory of reality [...], it would be double counting to list the fusion of the *xs* and also list the *xs*." (cf. Lewis 1991, p.81)

"While looking at one and the same external phenomenon, I can say with equal truth both "It is a copse" and "It is five trees", or both "Here are four companies" and "Here are 500 men". Now what changes here from one judgement to the other is neither any individual object, nor the whole, the agglomeration of them, but rather my terminology." (Frege 1960, §46, p.59); citation due to (Cotnoir 2014, p.4)

Defenders of the innocence thesis within the counting approach claim that counting all uncomposed entities within a mereological universe leads to the same result as [400] counting all entities within that universe (cf. Baxter 1988b, p.200). Opponents of the thesis claim that counting of uncomposed entities leads to a different result as counting all entities, if there are at least two different uncomposed entities at all (cf. Inwagen 1994, p.213; and Yi 1999, p.142). So, both say that mereology is ontologically innocent if it does not influence counting and that is to say that it is compatible with all counting results. If we take 'A' as a predicate for characterising uncomposed (atomic) entities, we can formulate this claim in the following way:

OIC A mereology M is ontologically innocent in the counting approach iff for every $n \geq 1$ $M \cup \{\exists_n^n x A(x)\} \cup \{\exists_n^n xx = x\}$ is consistent.
Thesis: Mereology is ontologically innocent in this sense.

So, assuming the existence of n atomic entities within a universe and starting up the abstraction machinery of an ontologically innocent mereology does not exceed the boundaries of the universe. The claim, e.g., that according to a mereology M the composition of two entities exists, does not dispense ontological innocence from M. Only the additional claim that according to M the composition is a different third entity characterises M as ontologically blameworthy, regardless if there are unconditioned existential assumptions (about a zero atom etc.) or not.

Two remarks seem to be helpful for understanding this thesis. Firstly, we set up the condition that $n \geq 1$, because $\exists_0^0$ in $\exists_0^0 x\varphi[x]$, contextually definable as $\exists x(x \neq x \;\&\; \forall y(\varphi[y] \leftrightarrow y = x))$, is invalid in classical logic. Note that this formula is contingent in free logic and that exactly this difference, viz. to state contingently $\exists_0^0 xx = x$, is mostly stressed as the non-logical ontological blameworthiness of classical logic by the representatives of free logic. Nevertheless we ignore this detail in our stipulation for reasons of simplicity.

Secondly, according to our use of language an infinite mereology, that is a mereology with some axioms of countable infinity and also some nominalistic mereologies, e.g. a mereology with the nominalistic principle of Nelson Goodman (cf. Goodman 1956, sect.2), "no distinction of entities without distinction of content" or, even shorter: "no creation without individuation", are ontologically blameworthy: the former inasmuch as such a mereology would be inconsistent with every finitary claim; the later inasmuch as a nominalistic mereology in the sense of Goodman is only consistent with the claim of the existence of individuals; and this is to claim that exactly $n = 2^a - 1$ entities exist, given that a uncomposed individuals exist. Again, our stipulation is discussable regarding adequacy, but for the purpose of our paper we can leave such a discussion aside by excluding infinite and nominalistic mereologies from our domain of discourse.

Furthermore, it should be mentioned that the literature on the innocence thesis or the thesis of *composition as identity* is rapidly growing. Several different innocence theses are discussed in that literature. It is an aim of this paper to consider mainly plural predication and by this, stick to standard first-order logic (FO) as background theory, whereas most other investigations start directly from the theory of plural quantification. E.g. in (Spencer 2013, p.1178) the so-called *strong composition as identity* thesis is discussed which claims that (necessarily) y is the composition of the xx (plural variable) iff $xx = y$. Although in the next section we are going to briefly discuss innocence theses in such a framework, we will stick mainly to the theses [401] formulated above, having primarily plural predication and not plural quantification (reference) in mind. The innocence theses discussed here should shed some light on the previously undiscussed possibility of understanding *composition as identity*.

We now begin our exact discussion of the innocence theses. First we will discuss shortly an approach to the innocence theses by help of plural quantification. Then we will go on with theories of plural predication, expand them in a second step by mereological definitions and axioms, and finally discuss their compatibility with the innocence theses.

3 Plural Quantification

Lewis himself suggested using the framework of plural quantification for defining mereological operations (cf. Lewis 1991, sect.3.2). To sum up the result regarding plural reference by plural quantification, one can say that the innocence theses hold if one accepts plural first-oder logic (PFO) as *logic*. PFO is, at first glance, a weak extension of standard first-order logic (FO) in the following way (cf. Linnebo 2003, sect.1):

- The vocabulary of FO is extended by the plurally refering variables $xx, yy, zz, \ldots$ and the two-place symbol '$\sqsubset$' (to be read as '... is one of the ...'; we take '$\sqsubset$' here just to abbreviate one specific descriptive two-place relation-symbol of FO).

- The set of language formation rules of FO is extended by: If φ is a formula and xx occurs freely in φ (henceforth: $\varphi[xx]$), then also $\forall xx \varphi[xx]$ and $\exists xx \varphi[xx]$ are formulae.

- The set of axioms of FO is extended by the following axioms PFO1–PFO6.

First, two axiom schemata on plural quantification analogous to the one of singular quantification:

PFO1 $\forall xx \varphi[xx] \rightarrow \varphi[xx/yy]$

PFO2 $\varphi[yy] \rightarrow \forall xx \varphi[yy/xx]$ (where $\varphi[yy/xx] = \varphi[yy]$)

Then an axiom and an axiom schema for the identity between plurals (this characterisation is similar to (Spencer 2013, p.1179)):

PFO3 $\forall xx\, xx = xx$

PFO4 $\forall xx \forall yy (xx = yy \rightarrow (\varphi[xx] \leftrightarrow \varphi[xx/yy]))$

Then an axiom schema on the non-emptyness of plural reference:

PFO5 $\forall xx \exists y\, y \sqsubset xx$

And finally the plural comprehension axioms instantiating the following schema—and stating the possibility of plural referring by non-empty complex properties:

PFO6 $\exists y \varphi[y] \rightarrow \exists xx \forall y (y \sqsubset xx \leftrightarrow \varphi[y])$

Now, it is just a small exercise to see that mereology can be embedded easily into PFO: One might define a mereological overlapping-relation $\square$ as follows:

PFO7 $xx \square yy \leftrightarrow \exists z(z \sqsubset xx \;\&\; z \sqsubset yy)$

[402] Since it is a theorem of PFO plus the definition above that:

T1 $\forall xx \forall yy \exists_1^1 zz \forall zz_1(zz_1 \square zz \leftrightarrow (zz_1 \square xx \vee zz_1 \square yy))$ ($\exists_1^1 \square$: PFO$_{\mathit{QED}}$)[1]

One can define a mereological operation for composition:

PFO8 $\textsc{sum}(xx, yy) = zz \leftrightarrow \forall zz_1(zz_1 \square zz \leftrightarrow (zz_1 \square xx \vee zz_1 \square yy))$

And for this composition-operation it holds that it is ontologically harmless in the sense that the composition of the *xxs* and *yys* is just the (*xx and yy*)*s*:

T2 $\forall xx \forall yy \forall zz(\textsc{sum}(xx, yy) = zz \leftrightarrow \forall x(x \sqsubset zz \leftrightarrow (x \sqsubset xx \vee x \sqsubset yy)))$ ($[\varepsilon/\sqsubset]$–modified OII: PFO$_{\mathit{QED}}$)

It is also innocent in the sense that it does not increase the number of elements in the domain of discourse. So, if there are n things that are one of the *xxs*, and if there are m things that are one of the *yys* (without being also one of the *xxs*), then there are also only $n + m$ things that are one of the (*xx and yy*)*s*—in case all things are one of the *xxs* or *yys*, the total number of things is $n + m$:

T3 $\forall xx \forall yy((\exists_n^n x\; x \sqsubset xx \;\&\; \exists_m^m y(y \sqsubset yy \;\&\; y \not\sqsubset xx)) \leftrightarrow \exists_{n+m}^{n+m} z\; z \sqsubset \textsc{sum}(xx, yy))$, furthermore: ($[A/\sqsubset]$–modified OIC: PFO$_{\mathit{QED}}$)

T4 $\forall xx \forall yy((\exists_n^n x\; x \sqsubset xx \;\&\; \exists_m^m y(y \sqsubset yy \;\&\; y \not\sqsubset xx) \;\&\; \forall x(x \sqsubset xx \vee x \sqsubset yy)) \rightarrow \exists_{n+m}^{n+m} z\; z = z$ ($[A/\sqsubset]$–modified OIC: PFO$_{\mathit{QED}}$)

Whether this way of abstraction is *de facto* ontologically harmless depends on the logical status of PFO. It was Boolos who first argued for the logicality of this framework and who also showed that an interesting part of second-order logic, namely the monadic part of it, can be interpreted in a more or less harmless way by allowing for plural reference (cf. Boolos 1984).

Critique against such a point of view is widespread. One of the strongest opponents is Charles Parsons, who extended Willard van Orman

[1] We are considering several systems of plural reference in this paper. In order to make transparent which consequences follow from which system, we always state in brackets the systems according to which a theorem is valid. 'QED' refers to a sketch of a proof in the appendix.

Quine's criterion for figuring out a theory's existential assumptions: "To Be is to be the Value of a Variable" (cf. Quine 1966, §37) in a way opposing directly Boolos' "To Be is to be a Value of a Variable (or to be Some Values of Some Variables)": Taking Quine's criterion for FO and expanding it to PFO does not lead to Boolos' interpretation, according to Parsons, but to a commitment of plural entities or collections, since the quantifiers of PFO range over such objects (cf. Parsons 1982). It is well known that the adequacy of Parsons' and Boolos' extension of Quine's original criterion depends on the background- or meta-theory one has in mind: If the meta-theory allows also for plural referring, then Boolos can uphold his interpretation by plurally referring to objects (of only seemingly collections). If not, then one has to join Parsons.

Another—very recent—critique against plural quantification as a tool for achieving ontological innocence stems from Graham Priest. Priest argues that quantifying in intensional contexts—especially into contexts of intentions—is *prima facie* singular quantification since "intentionality is normally defined as a mental state that is [403] focused on *an* object" (cf. Priest 2014, chpt.6.10). So, e.g., it makes perfect sense to only state that one is thinking of the *even numbers* if one assumes a framework of singular quantification, but not of plural quantification since "one cannot have an infinite number of mental foci" (cf. Priest 2014, p.95). Regarding contexts with intentions he thinks that one cannot get rid of non-plural fusion because one has to interpret such statements as statements about single objects (of intention), i.e. fusions that one singularly refers to (cf. Priest 2014, p.96).

Priest himself proposes a solution to the problem of *unrestricted composition* by pardoxical identification as follows:

1. He takes monadic second order paraconsistent logic (logic of paradox: *LP*) and defines a first order relation of identity by Leibniz' law.

2. Due to the non-transitivity of the paraconsistent biconditional, *LP*-identity ($=_{LP}$) turns out to be reflexive and symmetric, but not transitive.

3. Then Priest explains the unity of entities of a theory by expanding it to a so-called "gluon-model". The idea is that entities are unified by so-called "gluons", i.e. objects that have contradictory properties (being an object and being none; being part of a unity and being not part of it). With such objects one can, according to Priest, then try to unify an object by paradoxical identification:

- Take, e.g., a theory T with a domain $D = \{x, y, z\}$ where it holds that $x =_{LP} \textsc{sum}(y, z)$. Then one can *unify* x in Priest's framework by *identifying* it with its parts in an expanded gluon-model:
- T^G with $D^G = \{x, y, z, g\}$, where g is a *gluon*, glueing x and y together as well as x and z. The explanation of the unity of x with y and z starts no infinite glueing-regress (asking for further gluons g', g'', glueing x and g and g and y via g' and x and g and g and z via g'' together) since g is also considered to be no object and by this also to be no part of x that would be in need of an explanation for glueing. Within T^G it holds that (due to non-transitivity of LP-identity):
  - $\begin{array}{ccccc} x & =_{LP} & g & =_{LP} & y \\ & & \|_{LP} & & \\ & & z & & \end{array}$
  - But still $x \neq_{LP} z \neq_{LP} y$

  So, within this framework a fusion x is of course not identical with each single part y and z, but with the parts as a whole, i.e. the *gluon* that binds y and z together to x.
- Priest also proves an important result for his "theory of gluons": The expansion is conservative since all theorems of T are also theorems of T^G (cf. the conservation theorem in Priest 2014, chpt.2.10, especially 2.10.4).

The paraconsistent approach of Priest might be considered as a generalised solution to several semantic paradoxes (liar, predication, etc.) that also finds its application in the problem of unrestricted composition. Nevertheless, especially in its application to mereology, it is proven to be inflationary in the sense that all objects turn out [404] to have contradicting properties (cf. Casati and Fujikawa 2014, p.503). It is also a very unorthodox approach whose logical framework appears to some as "seriously weird, bizarre and pathological [because it] doesn't allow conditional proofs, [...] it not only tolerates, but is, inconsistent, and [...] it requires non-standard models" (cf. Pelletier 2015, p.825).

Regardless of the stance one takes here and also in the case of Parsons' argument, perhaps the most important critique against the "logical status of the theory of plural quantification" stems from Øystein Linnebo, who claims:

> Large parts of this theory can, without serious difficulties, be regarded as purely logical. This holds of the tautologies, the

axiom that says that pluralities are non-empty, and the natural deduction rules [here: PFO1–PFO5]. The only serious worry concerns the logical status of the plural comprehension axioms [here: PFO6. ...] The claim that all plural comprehension axioms are logical truths is a very strong one. In particular, this claim should not be confused with the weaker claim that the plural comprehension axioms are *true* [... which] is indeed rather plausible. (cf. Linnebo 2003, p.75)

Linnebo provides a convincing argument against the logicality thesis of PFO by showing that PFO with non-distributive predication is theoretically very strong inasmuch as a bulk of set-theoretical principles can be reconstructed in non-distributive PFO. Non-distributive predication is predication of properties to valuations of plural variables $xx, \ldots$ which is not reducible to predication of properties to valuations of individual variables $x, \ldots$ (cf. Linnebo 2003, p.79). So, if one can show for an open formula $\varphi[xx]$ that within the theory under consideration $\varphi[xx/x]$ is equivalent to $\forall x(x \sqsubset xx \leftrightarrow \varphi[xx/x])$, then the predication of φ to the valuation of xx is distributive.

As Linnebo concludes, due to the strength of PFO with non-distributive predication it seems so that it cannot be considered as purely *logical*.

As the definitions PFO7 and PFO8 on overlapping and SUM show, our embedding of mereology into PFO presupposes non-distributive predication. Since SUM is defined for plurals via $\circ$ and the latter is also defined for plurals via $\sqsubset$, the substitution of x for xx in $\text{SUM}(xx, yy) = zz$ would bring about an ill-formed expression containing '$\sqsubset x$' (even if one makes technical sense for individual variables flanking $\sqsubset$ it is to be supposed that no innocence thesis will hold). A much more general argument of Theodore Sider shows that an identification of the composition (referred to singularly) with its parts (referred to plurally) enforces in relevant cases non-distributivity (cf. Sider 2007, sect.3.1, especially p.8). Sider's innocence thesis (cf. Sider 2007, p.8) might be levelled down to the binary-SUM-case as follows—we also keep plurals and individuals in identity statements separated:

Strong composition as identity:
$\forall xx \forall yy \forall zz(\forall x(x \sqsubset zz \leftrightarrow (x \sqsubset xx \vee x \sqsubset yy)) \rightarrow \text{SUM}(xx, yy) = zz)$.

Strong composition as identity is just one direction of the innocence thesis/theorem T2. As mentioned above, due to language restrictions our formulation differs from that of Sider in that he refers to the composition singularly (with an individual variable) and we do so plurally (with a plural variable). Still, Sider's non-distributivity argument [405] also transmits to our embedding of mereology into PFO: For all cases where $\varphi[xx]$, $xx = yy$ and at least one $y \sqsubset yy$ is $\sim\varphi[xx/y]$ then T2 ("superstrong composition as identity"—(cf. Sider 2007, p.9)) enforces non-distributivity of φ. As an example take figure 1, where xx and yy contain all the same entities, just differently arranged. Also suppose that all the assumptions mentioned before hold and φ is to be interpreted as '. . . is rectangular'. Now suppose distributivity of φ. Then one can correctly infer $\varphi[xx/x_1]$ as well as $\varphi[xx/x_2]$. By T2 we get $xx = sum(xx_1, xx_2) = sum(yy_1, yy_2) = yy$ (take exactly $x_1 \sqsubset xx_1$, $x_2 \sqsubset xx_2$ etc.). By PFO4 (substitutivity of plural identicals) we can also correctly infer $\varphi[yy]$. But, again by distributivity of φ, we can infer incorrectly: $\varphi[yy/y_1]$ as well as $\varphi[yy/y_2]$. Hence φ cannot be distributive.

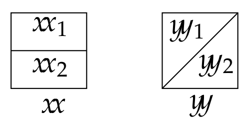

Figure 1: Example of enforcing non-distributivity by OII

Regarding our embedding it generally holds that the innocence theses enforce non-distributivity in most of the cases. And since non-distributivity is not guaranteed to be purely logical/ontologically harmless, our embedding of mereology into PFO is also not guaranteed to be purely logical, i.e. ontologically harmless.

One might think that there are more general ways of plurally referring. In the rest of the paper we will discuss this question with respect to theories of plural predication. We will show that such an alternative approach to the theses of ontological innocence also lacks plausibility.

4 Plural Predication

As we have already mentioned, there are different forms of predication, that is: different, at least partly, adequate interpretations of the 'is' of predication of our natural languages. In the following we will elaborate four more or less common theories which allow us to demonstrate that one has to give different answers to the question whether the thesis in OII or OIC holds, depending on the underlying theory of predication.

We begin with an axiom that is supposed to be an axiom of all four theories. Formally speaking it is just an abstract version of the axiom of extensionality of set theory based on FO without identity. We use it here because it allows us to build up easily a theory of identity based on predication only:

AP0 $\forall x \forall y(\forall z(z \varepsilon x \leftrightarrow z \varepsilon y) \rightarrow \forall z(x \varepsilon z \leftrightarrow y \varepsilon z))$

Both, a nominalistic as well as a platonistic interpretation of AP0 seem to be acceptable. The later by interpreting 'ε' set theoretically as element relation. The former by holding that if all entities that are one of x are also one of y and vice versa, then each of x is also one of those entities that each of y is one of and vice versa.

Let us now come to the first theory of predication, characterising 'ε' with the help of the following axiom:

AP1 $\forall x \forall y(x \varepsilon y \leftrightarrow x \in y)$

[406] As one can see, AP1 serves for the usual interpretation of 'is' in formal languages, provided we assume that for '$\in$' hold exactly all usual set theoretical axioms, specifically those of Zermelo-Fraenkel set theory. To say that Iseult is beautiful is according to AP1 modelled by saying that Iseult is a member of the set of all entities that are beautiful. Obviously for this form of singular predication (ε) hold all theorems of set theory ($\in$).

Of course some people will counter that a theory based on AP1 cannot be esteemed as a mereology. But despite the fact that Lewis counts his approach of constructing set theory on the basis of the parthood relation, the composition operation and the primitive operation of building up single sets as an approach within the mereological programme (cf. Lewis 1991, chpt.4), we stand in for the view that the set theoretical framework may be also part of a mereology. That is especially to say that one may not only build up mereologies within a syntactical logical framework, e.g. within a calculus of FO, but also within a semantic logical framework, that is set theory.

The second theory of predication we introduce is a part of Leśniewski's theory of predication and consists besides AP0 of the following axiom (for a much more elaborated investigation of Leśniewski's theory of plural predication and a bibliography see, e.g., (Simons 1981)):

AP2 $\forall x \exists y\, y \varepsilon x \,\&\, \forall x \forall y(x \varepsilon y \leftrightarrow (\exists z\, z \varepsilon x \,\&\, \forall z_1 \forall z_2(z_1 \varepsilon x \,\&\, z_2 \varepsilon x \rightarrow z_1 \varepsilon z_2) \,\&\, \forall z(z \varepsilon x \rightarrow z \varepsilon y)))$

The second conjunct of AP2 is the main axiom of Leśniewski's theory of predication, called 'Ontology' (cf. Betti 2010, p.311; and Ridder 2002, sect.1.2). There are two further axioms of Leśniewski's theory, whereof AP0 is the only consequence we need here. We have added the first conjunct, because we are going to talk only of entities that exist in fact.

Leśniewski thought that his theory of predication captures all relevant features of the latin expression 'est' or the polish predication-particle 'jest'. In agreement with his theory it holds that if someone states that Iseult is beautiful, then she states also that there is an entity that is Iseult, that every entities that are Iseult are each other and that every entity that is Iseult is one of the entities that are beautiful. So, according to him it is adequate only to state of exactly one entity that it is also one of some other entities. The first conjunct may be called 'existence condition', the second one 'uniqueness condition' and the third one 'condition of inclusion'. One can easily construct a set theoretical semantics for 'ε' which is adequate with respect to AP2, if one interprets 'ε' as single inclusion ($x\ {}_1{\subseteq}_{1+}\ y$ iff $x \subseteq y$ and $|x| = 1$). This set-theoretical interpretation is not necessary, but helpful regarding the theorems we give later on.

The third theory of predication is a just slightly modified form of AP2. We do not suppose the uniqueness condition within a predicational statement:

AP3 $\forall x \exists y\ y\varepsilon x\ \&\ \forall x \forall y (x\varepsilon y \leftrightarrow (\exists z\ z\varepsilon x\ \&\ \forall z(z\varepsilon x \rightarrow z\varepsilon y)))$

This condition within a predicational statement in the sense of AP2 is given up here in the following way: To say that Iseult is beautiful is to say that all entities that are Iseult are also one of the entities that are beautiful and that there is at least one entity that is Iseult. According to such a theory it is also adequate for predication to say of more than one entity that they are also one of some other entities. So, e.g., [407] to say that Tris and Iseult are beautiful is to say that all entities that are Tris and Iseult, that is Tris and that is Iseult, are also one of the entities that are beautiful. Although AP0 is already a consequence of AP3, we take for simplicity of notation both formulas to be axioms of the third theory of predication. Analogue to AP2 one can construct an adequate set theoretical semantics for AP3 if one interprets 'ε' as non-empty inclusion ($x\ {}_{1+}{\subseteq}_{1+}\ y$ iff $x \subseteq y$ and $x \neq \emptyset$).

The last theory of predication we introduce emerges if one keeps the existence condition and weakens the inclusion condition within a predicational statement in the sense of AP3 as follows:

AP4 $\forall x \exists y\ y\varepsilon x\ \&\ \forall x \forall y (x\varepsilon y \leftrightarrow \exists z (z\varepsilon x\ \&\ z\varepsilon y))$

Axiom AP4 is a very weak approach to predication and with respect to predication in natural languages surely not adequate. To claim that Tris and Iseult are beautiful is certainly not the same as claiming that one of them is also one of the entities that are beautiful, rather we are claiming that both are beautiful. Nevertheless, from a sceptical point of view regarding translation and investigation of reference, if anything at all, then AP4 seems to be acceptable as an adequate although not comprehensive statement about predication. Consider, e.g., a native that shouts 'gavagai' and points to a rabbit. If she claims 'gavagai ilhicha kano', then we may not be sure if she wants to say that all rabbits, the rabbit in front of us, the ears of the rabbit in front of us etc. are beautiful. Nevertheless we seem to be allowed for practical reasons to assume the very weak claim that something that is the ears of the rabbit in front of us or that is the rabbit itself or that is all rabbits etc. is also something that is beautiful. Again, AP0 is a consequence of AP4 but anyhow we take both formulas as axioms of the fourth theory of predication. An adequate set theoretical semantics for AP4 can be constructed with the help of the concept of a connecting set ($x \cap_{\neq \emptyset} y$ iff there is a z such that $z \cap x \neq \emptyset$ and $z \cap y \neq \emptyset$). It is easy to see that all concepts except of an empty concept can be brought into such a relation and by this gavagaian predication is really completely obscuring reference.

To sum up: We have postulated axioms for four theories of predication: the set theoretical, the Leśniewskian, a weakened Leśniewskian and a gavagaian theory of predication. It is easy to see that AP1, the specific axiom of the set theoretical theory of predication, is logically independent of all other specific axioms. One only has to substitute '$\in$' for 'ε' in AP2, AP3 and AP4 and sees that each of the substitution either is no theorem of set theory or even contradicts it. Also AP2 and AP3 are logically independent. Only AP2 and AP4 and AP3 and AP4 are logically connected as far as AP2 and AP3 are consequences of AP4.

In figures 2–5 the different theories of predication are illustrated. Whereas in singular predication each expression refers to exactly one object, in Leśniewski's theory of predication general expressions (predicates) can refer plurally, whereas singular expressions (subject terms) always refer singularly to objects. A weakened form of plural predication (figure 4) allows also subject terms to refer plurally to objects. It is here where the innocence thesis starts to make some sense: Replace, e.g., 'Tris and Iseult' by 'The composition of all red things' and replace 'married' by 'all red things', then one may argue that the composition of all red things simply are/is all [408] red things. Finally, *gavagaian* predication in figure 5 turns out to verify a categoric statement if something that is the subject is also something that is the universal.

Let us now state some properties of these theories of predication. While

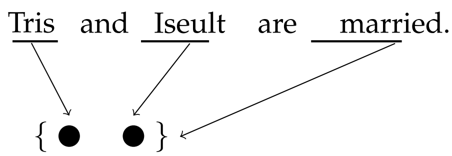

Figure 2: Sg. predication: AP1

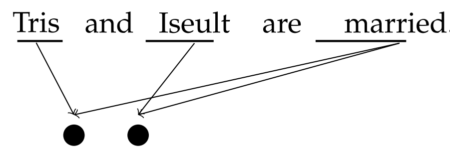

Figure 3: Leśniewskian pred.: AP2

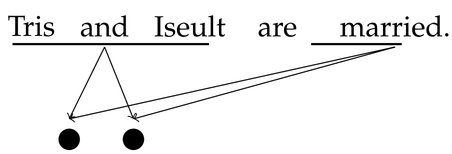

Figure 4: Weakened L. pred.: AP3

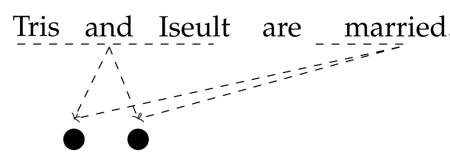

Figure 5: *Gavagaian* pred.: AP4

singular predication (AP1) is irreflexive and asymmetric, plural predication (AP2–AP4) is just not symmetric. Plural predication in the sense of Leśniewski is also not reflexive, because it makes on his view no sense to state of more entities that they are something. Plural predication in the weakened version and in the gavagaian sense is reflexive:

T5 $\forall x(x \varepsilon x)$ (Reflexivity: AP3–AP4)

All forms of plural predication are transitive:

T6 $\forall x \forall y \forall z(x \varepsilon y \,\&\, y \varepsilon z \rightarrow x \varepsilon z)$ (Transitivity: AP2–AP4)

Each theory of predication is strong enough to build up a theory of identity. For this purpose the classical definition of identity of set theory is sufficient:

DI1 $x = y \leftrightarrow \forall z(z \varepsilon x \leftrightarrow z \varepsilon y)$

It is clear that the so defined identity relation is an equivalence relation according to all four theories of predication:

T7 $\forall x\, x = x$ (Reflexivity: AP1–AP4)

T8 $\forall x \forall y(x = y \rightarrow y = x)$ (Symmetry: AP1–AP4)

T9 $\forall x \forall y \forall z(x = y \,\&\, y = z \rightarrow x = z)$ (Transitivity: AP1–AP4)

With the help of AP0 and DI1 we can show that there holds also the principle of *indiscernibility of identicals* with respect to predicational statements for all four theories (‘$\varphi[x]$’ and ‘$\varphi[x/y]$’ can be substituted by any formula built up of ‘ε’ and any expression that is definable with the help of ‘ε’ only):

T10 $\forall x \forall y(x = y \rightarrow (\varphi[x] \leftrightarrow \varphi[x/y]))$ (Leibniz’ Law I: AP1–AP4 ⓟ)

By providing reflexivity (T7) and indiscernibility (T10) definition DI1 based on each theory of predication is strong enough for identity. That it is not too strong, but exactly adequate, can be shown if one accepts a variant of Leibniz’ law of the *identity of indiscernibles* as condition of adequacy: embedded into the theories of predication it is stating that if all predicational statements about x and y are equal in truth, then x and y are identical:

T11 $\forall x \forall y(\forall z(x\varepsilon z \leftrightarrow y\varepsilon z) \rightarrow x = y)$ (Leibniz’ Law II: AP1–AP4)

[409] Regarding plural predication people sometimes distinguish two types of identity: identity between entities that are plurally referred to in predication, as, e.g., used in the claim that Tris and Iseult are identical with the lovers of the corresponding medieval epic. And identity between entities that are univocally referred to in predication, as, e.g., used in the claim that Iseult is identical with King Mark’s wife. Our definition is also a combination of both. Identity between plurals, sometimes also called ‘many-many identity’, is as just given in DI1. Singular identity, sometimes also called ‘one-one identity’ (cf. Baxter 1988a, p.577), comes in two variants:

T12 $\forall x \forall y(x = y \leftrightarrow \forall z(x\varepsilon z \leftrightarrow y\varepsilon z))$
(One-One Identity I: AP1–AP4)

T13 $\forall x \forall y(x = y \leftrightarrow x\varepsilon y \,\&\, y\varepsilon x)$ (One-One Identity II: AP3–AP4)

That there is no hybrid identity—sometimes also called ‘many-one identity’ (cf. Baxter 1988a, p.577)—is indicated by the following theorem:

T14 $\forall x \forall y(\exists^n_n z\, z\varepsilon x \,\&\, x = y \rightarrow \exists^n_n z\, z\varepsilon y)$ for any $n \geq 1$
(Identity is Non-Hybrid: AP1–AP4)

So, if some x that exactly n entities are is identical with y, then also exactly n entities are y. That is especially: there is no identity relation between one entity and many.

Up to now we have introduced four theories of predication:

- $\{$AP0,AP1,DI1$\}$: the set theoretical theory of singular predication, interpretable with the help of the element relation $\in$.
- $\{$AP0,AP2,DI1$\}$: the Leśniewskian theory of plural predication, interpretable with the help of the single inclusion relation ${}_{1}\subseteq_{1+}$.
- $\{$AP0,AP3,DI1$\}$: the weakened Leśniewskian theory of plural predication, interpretable with the help of the non-empty inclusion relation ${}_{1+}\subseteq_{1+}$.
- $\{$AP0,AP4,DI1$\}$: the gavagaian theory of plural predication, interpretable with the help of the relation of being connectable by a set $\cap_{\neq\emptyset}$.

In the next section we will have a look on the interrelation between these theories, mereology and the innocence thesis in OII. For brevity we make reference to each theory of predication by its specific axiom only (AP1–AP4). [410]

5 Mereology and Identification

All four theories of predication allow us separately to define basic mereological concepts: '$\preceq$' for the relation of being a part, '$\prec$' for the relation of being a proper part, '$\circ$' for the relation of overlapping and 'A' for being uncomposed, that is being atomic. Here are the definitions:

DM1 $x \preceq y \leftrightarrow \forall z(z\varepsilon x \rightarrow z\varepsilon y)$

DM2 $x \prec y \leftrightarrow x \preceq y \,\&\, x \neq y$

DM3 $x \circ y \leftrightarrow \exists z(z\varepsilon x \,\&\, z\varepsilon y)$

DM4 $A(x) \leftrightarrow \sim\exists y(y\varepsilon x \,\&\, y \neq x)$

One may wonder about DM3 and DM4, inasmuch as we use the predication relation and not the parthood relation for the definitions. But the reason for this is simple: According to AP1, 'ε' is interpreted as $\in$ and hence parthood defined in DM1 is understood as inclusion. If we had defined overlapping and being atomic in DM3 and DM4 with the help of the parthood relation, then, because of the empty set, everything would overlap everything and nothing would be atomic. Our definitions provide within

AP1 the usual set theoretical counterparts of mereology: inclusion for parthood, proper inclusion for proper parthood, non-empty cut for overlapping and being an urelement or an entity of type 0 for being atomic. That for AP2–AP4 our definitions are as usual can be seen by the following theorems:

T15 $\forall x \forall y(\exists z(z \varepsilon x \;\&\; z \varepsilon y) \leftrightarrow \exists z(z \preceq x \;\&\; z \preceq y))$ (AP2–AP4)

T16 $\forall x(\sim\exists y(y \varepsilon x \;\&\; y \neq x) \leftrightarrow \sim\exists y\, y \prec x)$ (AP2–AP4)

The, in DM2 defined, relation of being a proper part bears the usual properties of a partial order:

T17 $\forall x(x \not\prec x)$ (Irreflexivity: AP1–AP4)

T18 $\forall x \forall y(x \prec y \rightarrow y \not\prec x)$ (Asymmetry: AP1–AP4)

T19 $\forall x \forall y \forall z(x \prec y \;\&\; y \prec z \rightarrow x \prec z)$ (Transitivity: AP1–AP4)

As one can imagine, plural predication mostly coincides with parthood:

T20 $\forall x \forall y(x \varepsilon y \rightarrow x \preceq y)$ (Predication implies Parthood: AP2–AP4)

T21 $\forall x \forall y(x \preceq y \rightarrow x \varepsilon y)$ (Parthood implies Predication: AP3–AP4)

Up to now the mereological extension of the four theories of predication were just definitional. That the definitional extensions are formally adequate is mainly shown with the help of the theorems T17 and T19 on the irreflexivity and transitivity of the proper parthood relation. The core of all mereologies is not definitional and non-conservative axiomatic, namely the principle of composition. Here it is:

AM1 $\forall x \forall y \forall z(sum(x,y) = z \leftrightarrow \forall z_1(z_1 \circ z \leftrightarrow (z_1 \circ x \vee z_1 \circ y)))$

Also for *sum*-formulas the Leibniz laws hold. It can be shown that the creative part of this extension with respect to AP2–AP4 and the mereological definitions DM1–DM4 is exactly the existence condition of composition (uniqueness is already a consequence of DM3 and one of AP2–AP4): [411]

T22 $\forall x \forall y \exists z \forall z_1(z_1 \circ z \leftrightarrow (z_1 \circ x \vee z_1 \circ y))$
(Creativity of Unrestricted Composition: AP1–AP4)

This is to state for any two entities the existence of their composition. Because of this reason axiom AM1 is sometimes called 'the principle of *unrestricted composition*'. Composition as defined here bears the usual properties, e.g. that the order while doing composition is irrelevant:

T23 $\forall x \forall y sum(x, y) = sum(y, x)$ (Commutativity: AP1–AP4)

T24 $\forall x \forall y \forall z sum(sum(x, y), z) = sum(x, sum(y, z))$ (Distributivity: AP1–AP4)

Or that there is no "hocus-pocus" in single composition:

T25 $\forall x sum(x, x) = x$ (Innocence of Selfcomposition: AP1–AP4)

But, as some people would put it: is there also no "hocus-pocus" in general, that is, as we would like to put it: holds the thesis OII? Well, it is easy to figure it out: we use 'M' for the set containing the mereological definitions DM1–DM4 and axiom AM1. It can be shown that:

T26 AP1∪M contradicts the thesis in OII. (QED)

T27 AP2∪M is compatible with the thesis in OII. (QED)

T28 AP3∪M is compatible with the thesis in OII. (QED)

T29 AP4∪M entails the thesis in OII. (QED)

There is even a stronger result: it can be shown that stating that the composition of x and y is identical with x and y (plurally referred to) forces one to gavagaian predication:

T30 The union of {AP0,DI1}∪M and the thesis in OII entails AP4. (QED)

The more general result concerns the question whether mereology is ontologically innocent within the identification approach. And our answer is: well, it depends on your theory of predication. Using singular predication: no, it is not. Using plural predication: yes, it is, but only if you have some kind of *gavagaian* theory of plural predication.

Stating the thesis in OII is compatible with plural predication, but it entails the very sceptical gavagaian theory. So let us have a look on the second version of the innocence thesis and figure out, if stating only the second version is less restrictive.

6 Mereology and Counting

For all cases, except one, it is possible to construct a, between the composition and its parts, differentiating relationship. The only exceptional case is the one assuming the existence of exactly one entity. Within this case there is no differentiating relationship (T25). But for all other cases there is one, e.g. that of being a part of one's part, or not reflexively put: the relationship of being an improper part of an atom. So, assuming the existence of exactly two or more mereological atoms (e.g. $\exists_2^2 x A(x)$) and performing abstraction with the help of the abstraction machinery of mereology is incompatible with claiming that there exist two or more entities at all [412] ($\exists_2^2 x x = x$). Let us state this point even more explicitly (similar to our argument in the characterisation of OII):

1. Assume the existence of exactly two atoms: $\exists_2^2 x A(x)$
2. And assume the innocence thesis within the counting approach (OIC).
3. According to the usual counting method the first assumption is to be spelled out as $\exists x \exists y (x \neq y \,\&\, \forall z (A(z) \leftrightarrow z = x \vee z = y))$.
4. By *unrestricted composition* (AM1) and the definition of being uncomposed (DM4) it follows: $\exists x \exists y \exists z\; z = sum(x, y)$ and hence $\exists z \sim A(z)$.
5. But then, because of 3. and 4., it holds: $\exists x \exists y \exists z (x \neq y \,\&\, z \neq x \,\&\, z \neq y)$.
6. And this contradicts the claim that there are exactly two entities at all which is according to the usual counting method spelled out as $\exists x \exists y (x \neq y \,\&\, \forall z (z = x \vee z = y))$ and which follows from 1. and 2..

Opponents of the innocence thesis blame assumption 2., that is the innocence thesis within the counting approach, for the contradiction in 5. and 6.. Defenders of the innocence thesis refute 6. itself. But how can one accept both assumptions and yet refuse one of their consequences? It is just by refuting an intermediate step, more precisely: by refuting the spelling out with the help of the usual counting method.

There are two main strategies of the defenders. One is to refute an application of the usual counting method for counting entities per se and one is to allow such an application, but apply within the innocence thesis OIC an unorthodox method of counting. Let us have a short look on the first strategy!

A motivating text passage for this strategy is the following one:

> Consider the express check-out line in a grocery store. It says 'six items or less'. You have a six-pack of orange juice. You might well wonder if you have one item or six items. But you would never hesitate to go into the line for fear of having seven items: six cans of orange juice plus one six-pack. (Baxter 1988b, p.200)

According to Donald L. M. Baxter there are different methods of counting and if one ascribes ontological blameworthiness to mereology, then she is mixing up these methods in an illegitimate way. There are many examples for different methods of counting in everyday life. For mereology a distinction of counting atoms and counting composed entities is relevant. To avoid troubles of counting composed entities that are parts of composed entities, we will concentrate on counting composed entities that are only parts of themselves. For this purpose we give a definition for such universal entities:

DM5 $U(x) \leftrightarrow \sim A(x) \,\&\, \forall y(x \preceq y \rightarrow x = y)$

The method for counting atoms was already introduced in our argument above (3.). Generally, for counting entities that have a property φ, we use the following method:

$$\begin{aligned} &\exists^n_n x\varphi[x] \text{ can be contextually defined by} \\ &\exists x_1 \ldots \exists x_n([\&_{1\leq i<j\leq n} x_i \neq x_j] \,\&\, [\&_{1\leq i\leq n} \varphi[x_i]] \,\&\, \\ &\qquad \forall y(\varphi[y] \rightarrow [\vee_{1\leq i\leq n} y = x_i]) \end{aligned}$$

As far as defenders of the innocence thesis with this strategy think that it is counting entities twice if one counts both, the atomic and the composed entities, they are [413] willing to reject each counting method that counts atoms and universals together. So, e.g., they agree with setting up a counting method for atoms ($\exists^n_n xA(x)$) and also with setting up a counting method for universals ($\exists^n_n xU(x)$). But since counting selfidenticals ($\exists^n_n xx = x$) is counting atoms and universals together ($\forall x(U(x) \vee A(x) \rightarrow x = x)$

is obviously logically valid), they reject this counting method. In this sense OIC is vacuously true, insofar as mereology is compatible with counting atoms; and counting selfidenticals is illegitimate and hence need not to be considered in OIC.

This strategy may be seen as helpful in view of ordinary language discourse and as a good tool for making technical sense of some at first glance pardoxical arguments. But of course it comes at high costs, namely giving up the orthodox method of counting and that is giving up some parts of classical logic. So it is more than desirable to find an alternative. Such an account is given by the second strategy.

A hint for this strategy is to be found in the following citation:

> Australia and New South Wales are not identical, but they are not completely distinct from each other. They are partially identical, and this partial identity takes the form of the whole-part 'relation' [...] Partial identity admits of at least rough-and-ready degree. Begin with New South Wales and then take larger and larger portions of Australia. One is approaching closer and closer to complete identity with Australia. (cf. Lewis 1991, pp.82f)

The main idea is that a composed entity is partially identical with each of its proper parts and fully identical with all of them together. Partial identity is to be understood as overlapping inasmuch two entities x and y are partially identical means that they have identical parts in common. One can easily introduce a comparative concept of partial identity (x is more partially identical with z than y as, e.g., used in 'East-Australia is more partially identical with Australia than New South Wales is.') or even—as indicated in the citation—a quantitative one (x is to the degree of n/m partially identical with y as, e.g., used in 'New South Wales is to the degree of 1/7 partially identical with Australia.') by counting (atomic) parts, which is no problem in the finite case. But for our purpose it is enough to stick to the qualitative concept:

DM6 $x =_p y \leftrightarrow x \circ y$

Partial identity would suffice for defining the parthood relation and is also a necessary condition for a theory of identity:

T31 $\forall x \forall y(x \preceq y \leftrightarrow \forall z(z =_p x \rightarrow z =_p y))$ (Parthood: AP1–AP4)

T32 $\forall x \forall y(x = y \rightarrow \forall z(z =_p x \leftrightarrow z =_p y))$ (Identity: AP1–AP4)

But of course partial identity is far away of being sufficient for a full theory of identity: it is no equivalence relation as far as transitivity fails and because of this also the principle of *indiscernibility of identicals* fails. Nevertheless one can make sense of partial identity for counting by providing a counting method that advices one to [414] distinguish entities not strictly but by partial non-identity, that is disjointness. One only has to substitute '$=_p$' for '$=$' in the abstract counting method given above:

$$\begin{aligned} &\exists\!\!\exists^n_n x\varphi[x] \text{ can be contextually defined by} \\ &\exists x_1 \ldots \exists x_n([\&_{1\le i<j\le n} x_i \neq_p x_j] \,\&\, [\&_{1\le i\le n}\varphi[x_i]] \,\& \\ &\qquad \forall y(\varphi[y] \rightarrow [\vee_{1\le i\le n} y =_p x_i]) \end{aligned}$$

It is clear that two atoms are disjoint, that is, they are not partially identical:

T33 $\forall x \forall y(A(x) \,\&\, A(y) \,\&\, x \neq y \rightarrow x \neq_p y)$ (Atoms: AP1–AP4)

And because of this reason it is easy to show that applying the counting method above within OIC is in favour of OIC:

T34 $\exists\!\!\exists^n_n x A(x) \rightarrow \exists\!\!\exists^n_n x\; x = x$ holds for all $n \geq 1$ (OIC: AP1–AP4)

But one should note that this method of counting has a property that may be seen as feature or paradoxical: By claiming that there exist exactly n atoms in the mereological universe, with the help of the given counting method one does not only claim that there exist n entities within the universe at all (T34), but one also claims that there exists exactly one entity within the universe:

T35 $\exists\!\!\exists^n_n x A(x) \rightarrow \exists\!\!\exists^1_1 x x = x$ holds for all $n \geq 1$
(Counting the Universal: AP1–AP4)

So, this counting method leads to the feature or paradox of stating the existence of exactly n entities (parts) while stating also the existence of exactly one entity ("the parts counted as one thing"). Of course, dissolved according to our definitions it is nothing more than stating that there are n fully disjoint parts (atoms) while there is also one overall overlapping whole (universal).

The question arises why one should see exactly in the method of counting partially identicals as defined above the relevant method for counting regarding ontological innocence. Why should one not accept, e.g., the method of counting maternally identicals as the relevant method for counting, where x is maternally identical with y iff the mother of x is identical with the mother of y? Here comes the so-called 'weak composition thesis' of Lewis into play:

> Mereological relations [...] are strikingly analogous to ordinary identity. So striking is this analogy that it is appropriate to mark it by speaking of mereological relations—the many-one relation of composition, the one-one relations of part to whole and overlap—as kinds of identity. Ordinary identity is the special limiting case of identity in the broadened sense. (cf. Lewis 1991, p.83)

Lewis lists five respects in which he thinks that a striking analogy holds. We skip the non-logical spatio-temporal analogy and consider only the first four respects—assuming here the framework of free logic and assuming that functional symbols are introducable into theories only by definition, hence we assume that all theories include existence and uniqueness conditions explicitly as axioms (cf. Lewis 1991, sect.3.6): [415]

(i) "just as it is redundant to say that x and y exist when x is identical with y, so it is redundant to say that x and the *ys* exist, when x is a fusion of the *ys*."
1: Ex and $x = y$ entails logically Ey
2: Ex, Ey and $z = sum(x, y)$ entails mereologically Ez

(ii) "just as, given that x exists, it is automatically true that something identical with x exists, so, given that the *xs* exist, it is automatically true that a fusion of the *xs* exists."
1: Ex entails logically $\exists y(x = y \,\&\, Ey)$
2: Ex, Ey entails mereologically $Esum(x, y)$

(iii) "just as there cannot be two things both of which are identical with x, so there cannot be two things both of which are fusions of the *xs*. There is something analogous to the transitivity of identity in this feature of composition."
1: $x = z$ and $y = z$ entails logically $x = y$
2: $x = sum(z, w)$ and $y = sum(z, w)$ entails mereologically $x = y$

(iv) "just as fully to describe x is fully to describe the object that is identical with x, so fully to describe the *xs* is fully to describe their fusion."

1: $\varphi[x]$ and $x = y$ entails logically $\varphi[x/y]$, where $\varphi[x]$ is to be taken a representation of a full description of x
2: For establishing the analogical part within mereology, it seems to be necessary to qualify the statement, as e.g. Peter van Inwagen did by "[...so fully to describe the *xs* is] fully to describe the distribution of local properties [within their fusion]." (cf. Inwagen 1994, p.218):
$\varphi[x]$, $\psi[y]$ and $z = sum(x, y)$ entails mereologically $\forall w(w \prec z \rightarrow \varphi[x/w] \vee \psi[y/w])$

Analogies, perhaps would be striking for the project of weakening the theory of identity (as standards for ontological innocence) if there were no other theories that satisfy the analogies and are nevertheless clearly not acceptable for such standards. But as, e.g., Byeong-Uk Yi has shown, there are other theories satisfying these analogies which nevertheless are clearly not acceptable for standards of ontological innocence. One may take as example the accompaniment theory of Yi that satisfies the same structural claims as above—e.g., it is also redundant to say that x and y exists, when y accompanies x—but no one would accept the accompany-relation as a logical relation that resembles identity (cf. Yi 1999, pp.150ff). And for this reason Lewis' approach to single out some mereological relations as relevant for the innocence thesis in OIC seems to be inadequate.

7 Conclusion

Although the mereological composition principle may be a handy tool for categorisation, we have seen that the counter-arguments against an unrestricted composition principle cannot be overcome by simply stating an innocence thesis. Regarding the underlying theory of reference and predication there are four options available to argue for such a thesis: [416]

	sg. predication	**pl. predication**
sg. quantification	FO (AP1)	ε-theory (AP2–AP4)
pl. quantification	Distributive MSO	Non-distributive MSO

Regarding FO (first-order logic) none of the innocence theses introduced in section 2 is consistent. Distributive MSO (monadic second order logic) is too weak to embed mereology. Non-distributive MSO is ontologically blameworthy since it allows for the reconstruction of a bulk of set theory. And plural predication (ε) in the tradition of Leśniewski can be made fruitful for the innocence theses only if it is some kind of *gavagaian* predication, i.e. predication that completely obscures reference. Therefore, in order to keep the principles of classical predication and logical counting one is forced to give up the theses on the ontological innocence of mereology.

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Technical Appendix

Every fusion of a finite number of individuals $a_1, \ldots, a_n$ is due to commutativity and distributivity of *sum* easily performable by an iterative application of the two-placed composition operation. For the infinite case one would just need to add a name-forming operator ${}_{\Sigma}x$ similar to the iota-operator for descriptive descriptions. One may then define possibly infinite fusions contextually by ${}_{\Sigma}x\varphi[x] = y \leftrightarrow \forall z(z \circ y \leftrightarrow \varphi[x])$.

In the following proofs we make use of Boolos' result that PFO can be embedded into monadic second order logic (MSO) and vice versa with the following one-one mapping *tr* (cf. Linnebo 2003, p.74):

- $tr(x \sqsubset xx) = X(x)$
- $tr(\sim\varphi) = \sim tr(\varphi)$, $tr(\varphi \,\&\, \psi) = tr(\varphi) \,\&\, tr(\psi), \ldots$
- $tr(\exists x\varphi[x]) = \exists x\, tr(\varphi[x])$, $tr(\forall x\varphi[x]) = \forall x\, tr(\varphi[x])$
- $tr(\exists xx\varphi[xx]) = \exists X\, tr(\varphi[xx])$, $tr(\forall xx\varphi[xx]) = \forall X\, tr(\varphi[xx])$

To shorten MSO proofs we also make use of the result that MSO can be embedded into restricted Zermelo set theory, also called 'monadic second order Zermelo set theory', consisting of five axioms: extensionality, pairing, power set, union, infinity and separation (the latter is the only monadic formula: $\forall X \forall x \exists y \forall z(z \in y \leftrightarrow (z \in x \mathbin{\&} X(z)))$. For details see (Pollard 2015, chpt.VII.2–4).

Proof.

- Proof of T1: By definition PFO7 and the translation manual *tr* we get: $tr(\forall \mathbb{x} \forall \mathbb{y} \exists_1^1 \mathbb{z} \forall \mathbb{z}_1(\mathbb{z}_1 \square \mathbb{z} \leftrightarrow (\mathbb{z}_1 \square \mathbb{x} \vee \mathbb{z}_1 \square \mathbb{y})))$ is logically equivalent to $\forall X \forall Y \exists_1^1 Z \forall Z_1(\exists x(Z_1(x) \mathbin{\&} Z(x)) \leftrightarrow \exists x(Z_1(x) \mathbin{\&} (X(x) \vee Y(x))))$. The [417] uniqueness claim ($\exists^1 Z \ldots$) follows immediately by transitivity of the biconditional ($\leftrightarrow$). By restricted monadic second order comprehension (instantiation: $\exists Z \forall x(Z(x) \leftrightarrow (X(x) \vee Y(x)))$) we get $\exists_1 Z \ldots$.

- Proof of T2: By definitions PFO7 and PFO8 and *tr* one has to prove that $\forall Z_1(\exists x(Z_1(x) \mathbin{\&} Z(x)) \leftrightarrow \exists x(Z_1(x) \mathbin{\&} (X(x) \vee Y(x))))$ is equivalent to $\forall x(Z(x) \leftrightarrow (X(x) \vee Y(x)))$. The step from the latter to the former is up to applying finally PFO2 for generalizing Z_1 completely FO. A set-theoretical translation of the former would be: $z_1 \cap z \neq \emptyset$ iff $z_1 \cap (x \cup y) \neq \emptyset$, therefore $z_1 \cap z = \emptyset$ iff $z_1 \cap (x \cup y) = \emptyset$ (for all z_1). Now, suppose $z \neq x \cup y$. Then there is an $a \in z$ which is not in $x \cup y$: $a \notin x \cup y$ (or the other way round: $a \in x \cup y$, but $a \notin z$). Let $z_1 = \{a\}$. Then $z_1 \cap z = \{a\}$, whereas $z_1 \cap (x \cup y) = \emptyset$ in contradiction to the equivalence above (similarly for the other case). Hence $z = x \cup y$, which can be translated back to $\forall x(Z(x) \leftrightarrow (X(x) \vee Y(x)))$.

- Proof of T3: By definitions PFO7 and PFO8 and *tr* one has to prove $(\exists_n^n x X(x) \mathbin{\&} \exists_m^m y(Y(y) \mathbin{\&} \sim X(y))) \leftrightarrow \exists Z \exists_{n+m}^{n+m} z(Z(z) \leftrightarrow \forall Z_1(\exists w(Z_1(w) \mathbin{\&} Z(w)) \leftrightarrow \exists w(Z_1(w) \mathbin{\&} (X(w) \vee Y(w)))))$ which can be translated further into the monadic second order Zermelo set theoretically valid cardinality claim: $|x| = n$ and $|y \setminus x| = m$ iff $|z| = |x \cup y| = n + m$.

- Proof of T4: Follows immediately from T3 by the extra condition: For all x holds: $x \in X \cup Y$.

- Correctness of ε in AP2 interpreted as ${}_1{\subseteq}_{1+}$ ($x \,{}_1{\subseteq}_{1+}\, y$ iff $x \subseteq y$ and $|x| = 1$): One just has to prove that the following statements as set-theoretically valid: for all x exists a y: $y \,{}_1{\subseteq}_{1+}\, x$; for all x and all y: If $x \,{}_1{\subseteq}_{1+}\, y$, then there exists a z such that $z \,{}_1{\subseteq}_{1+}\, x$, and for all z_1, z_2: If $z_1 \,{}_1{\subseteq}_{1+}\, x$ and $z_2 \,{}_1{\subseteq}_{1+}\, x$, then $z_1 \,{}_1{\subseteq}_{1+}\, z_2$; furthermore: For all z: If $z \,{}_1{\subseteq}_{1+}\, x$, then $z \,{}_1{\subseteq}_{1+}\, y$. The existence of a singleton-subset is guaranteed by the power set axiom. Regarding the equivalence: ($\Rightarrow$)

Assume $x \,{}_{1}{\subseteq}_{1+}\, y$. Then we know $|x| = 1$ and by this $x \,{}_{1}{\subseteq}_{1+}\, x$, hence: There is a z: $z \,{}_{1}{\subseteq}_{1+}\, x$; since $|x| = 1$ we also get by extensionality that the element(s) of a singleton-subset of x are/is unique. And by transitivity of ${}_{1}{\subseteq}_{1+}$ we arrive at the inclusion condition. ($\Leftarrow$): Assume (i) there is a z such that $z \,{}_{1}{\subseteq}_{1+}\, x$, (ii) for all z_1, z_2: If $z_1 \,{}_{1}{\subseteq}_{1+}\, x$ and $z_2 \,{}_{1}{\subseteq}_{1+}\, x$, then $z_1 \,{}_{1}{\subseteq}_{1+}\, z_2$, (iii) for all z: If $z \,{}_{1}{\subseteq}_{1+}\, x$, then $z \,{}_{1}{\subseteq}_{1+}\, y$. Now assume that $x \,{}_{1}{\not\subseteq}_{1+}\, y$. Then there are three cases to be considered: $x \not\subseteq y$ or $|x| \neq 1$ or $y = \emptyset$. But by (i) we know that $|x| \geq 1$ and hence by (iii) we get: $y \neq \emptyset$. Furthermore, take x to be $\{x_1, x_2, \dots\}$. By (ii) we get: $x_1 = x_2$, $x_1 = x_3$, ... Hence $|x| = 1$. But now we also get by (iii) that $x \,{}_{1}{\subseteq}_{1+}\, y$, hence $x \subseteq y$. So $x \,{}_{1}{\subseteq}_{1+}\, y$ in general. (Correctness for AP0 is straightforward: $z \,{}_{1}{\subseteq}_{1+}\, x$ exactly when $z \,{}_{1}{\subseteq}_{1+}\, y$ for any z implies $x \,{}_{1}{\subseteq}_{1+}\, z$ exactly when $y \,{}_{1}{\subseteq}_{1+}\, z$ for any z and vice versa.)

- Correctness of ε in AP3 interpreted as ${}_{1+}{\subseteq}_{1+}$ ($x \,{}_{1+}{\subseteq}_{1+}\, y$ iff $x \subseteq y$ and $x \neq \emptyset$): Proof analogous to the correctness proof above (without making assumption (ii)).

- Correctness of ε in AP4 interpreted as $\cap_{\neq\emptyset}$ ($x \cap_{\neq\emptyset} y$ iff there is a z such that $z \cap x \neq \emptyset$ and $z \cap y \neq \emptyset$): One just has to prove that: There is a z $z \cap x \neq \emptyset$ and $z \cap y \neq \emptyset$ iff there is a z such that there is a z_1 $z_1 \cap z \neq \emptyset$ and $z_1 \cap x \neq \emptyset$, and there is a z_2 such that $z_2 \cap z \neq \emptyset$ and $z_2 \cap y \neq \emptyset$. ($\Rightarrow$): Follows immediately by iteration and piecewise existential generalization. ($\Rightarrow$): [418] Assume that x and y are connected via z_1 and z_2 respectively to z. Then x and y are connected directly via $z_1 \cup z_2$ and hence there is a z connecting x and y ($x \cap (z_1 \cup z_2) \neq \emptyset \neq y \cap (z_1 \cup z_2)$). (Correctness regarding AP0 is due to the symmetry of the connectable-relation.)

- Proof of T5–T9: Straightforward FO (regarding AP1: set-theoretical).

- Proof of T10: Since the only primitive expressions are ε and (later on in section 5) *sum*—all the other notions $=$, $\preceq$, $\prec$ etc. are introduced by definitions alone—for proving the *indiscernibility of identicals* we have to consider only formulas that contain ε and *sum* as non-logical expressions. We do so by induction on the complexity of formulas: Assume $x = y$ is valid in the theory, i.e. by definition DI1: $\forall z(z\varepsilon x \leftrightarrow z\varepsilon y)$ is valid in the theory. Then we have to consider the following inductive basis (degree of complexity of φ is 0): $\varphi[x]$ is valid and of the form:

1. $z_1 \varepsilon z_2$: In this case the substitution is idle, hence $\varphi[x] = \varphi[x/y]$ and hence $\varphi[x] \leftrightarrow \varphi[x/y]$ is valid.
2. $x \varepsilon z_1$: By AP0 we get $x \varepsilon z_1 \leftrightarrow y \varepsilon z_1$ and by this also $y \varepsilon z_1$.
3. $z_1 \varepsilon x$: With DI1 we get immediately $z_1 \varepsilon y$.
4. $x \varepsilon x$: With DI1 we get $y \varepsilon y \leftrightarrow y \varepsilon x$; with AP0 we get $x \varepsilon x \leftrightarrow y \varepsilon x$, and hence $y \varepsilon y$.
5. $x \varepsilon y$ or $y \varepsilon x$: Cf. case 4 ($x \varepsilon y[x/y] = y \varepsilon y = y \varepsilon x[x/y]$).
6. $sum(x, z_1) \varepsilon z_2$: By AM1 and DM3 we get $z \varepsilon z_2 \leftrightarrow \forall z_3(\exists z_4(z_4 \varepsilon z_3 \ \& \ z_4 \varepsilon z) \leftrightarrow \exists z_4(z_4 \varepsilon z_3 \ \& \ (z_4 \varepsilon x \vee z_4 \varepsilon z_1)))$; by the result on case 3 we get $z \varepsilon z_2 \leftrightarrow \forall z_3(\exists z_4(z_4 \varepsilon z_3 \ \& \ z_4 \varepsilon z) \leftrightarrow \exists z_4(z_4 \varepsilon z_3 \ \& \ (z_4 \varepsilon y \vee z_4 \varepsilon z_1)))$ and hence by DM3 and AM1 $sum(y, z_1) \varepsilon z_2$.
7. All the other cases of *sum* in φ are either analogous to 6 or follow immediately from basic features of *sum* (commutativity, distributivity, selfcomposition: T23–T25).

The induction step is straightforward FO.

- Proof of T11–T25: Straightforward FO.

- Proof of T26: Take the following model: $x = 1, y = 2$. Then, by AP1 and AM1, we get $z = sum(x, y) = \{1, 2\}$. According to OII $z = sum(x, y)$ iff $x \in z$, $y \in z$ and for all $z_1 \in z$: $z_1 \in x$ or $z_1 \in y$. But, although $x \in sum(x, y)$, neither $x \in x$ nor $x \in y$ (if it were the case that $x \in y$, then $sum(x, y) = y$ and by this it wouldn't be the case that $x \in sum(x, y)$). W.l.o.g. this holds for any model whose domain contains at least two objects. So OII and AP1 are incompatible in case that there are at least two objects.

- Proof of T27: Since both, AP2 and OII are consequences of AP4 and AP4 is consistent (see below), it follows also that AP2 and OII are consistent.

- Proof of T28: Analogous to the proof of T27.

- Proof of T29: (That AP2 and AP3 follow from AP4 is straightforward FO.) The proof of OII by help of AP4 and M is a quite long, but can be verified, e.g., by an automatic FO-prover (used here: *Prover9*). Using all the definitions the problem reduces to:

  – From AP4: $\forall x \exists y \ y \varepsilon x \ \& \ \forall x \forall y(x \varepsilon y \leftrightarrow \exists z(z \varepsilon x \ \& \ z \varepsilon y))$, and [419]

  - the creative part of M (translation of T22): $\forall x \forall y \exists z \forall z_1 (\exists z_2 (z_2 \varepsilon z_1 \,\&\, z_2 \varepsilon z) \leftrightarrow \exists z_2 (z_2 \varepsilon z_1 \,\&\, (z_2 \varepsilon x \vee z_2 \varepsilon y)))$
  - follows OII—translated as: $\forall x \forall y \forall z ((x \varepsilon z \,\&\, y \varepsilon z \,\&\, \forall z_1 (z_1 \varepsilon z \rightarrow (z_1 \varepsilon x \vee z_1 \varepsilon y))) \leftrightarrow \forall z_1 (\exists z_2 (z_2 \varepsilon z_1 \,\&\, z_2 \varepsilon z) \leftrightarrow \exists z_2 (z_2 \varepsilon z_1 \,\&\, (z_2 \varepsilon x \vee z_2 \varepsilon y))))$.

Also AP4 and M are consistent which is easily verifiable (used here: *Mace4*). Here is the relevant input and output for *Prover9* and *Mace4* (for the proof the programme ran about 20min on an ordinary machine):

```
%Assumptions:
%AP4
((all x(exists y(E(y,x))))&(all x(all y(E(x,y)<->(exists z(E(z,x)&E(z,y)))))))
&
%DM3: O for overlapping
(all x(all y(O(x,y)<->(exists z(E(z,x)&E(z,y))))))
&
%DI1
(all x(all y(I(x,y)<->(all z(E(z,x)<->E(z,y))))))
&
%AM1: S for sum (existence and uniqueness are built in)
((all x(all y(all z(S(x,y,z)<->(all z1(O(z1,z)<->(O(z1,x)—O(z1,y))))))))&(all x(all y(exists z(S(x,y,z)&(all z1(S(x,y,z1)->I(z1,z))))))))
.
%
%
%Goals:
%OII
(all x(all y(S(x,y,z)<->((E(x,z)&E(y,z))&(all z1(E(z1,z)->(E(z1,x)—E(z1,y))))))))
.
```

- Proof of T30: Similar reduction and method as used above:
  - From OII—translated as above—, and
  - the creative part of M (translation of T22)
  - follows AP4.

So, it turns out that under the assumption of unrestricted mereological composition OII and AP4 are equivalent, i.e. unrestricted composition and OII enforces *gavagaian* predication.

- Proof of T31–T35: Straightforward FO. Regarding T35 note that the not dissolved way of counting (by distinguishing only disjunct objects) is paradoxical but can be made coherent by adding to the basis case of the contextual definition for $\exists\!\!\exists_n^n$ the condition that the entity under consideration is atomic. I.e.: For [420] $\exists\!\!\exists_1^1 x\varphi[x]$ use $\exists x(A(x) \mathbin{\&} \varphi[x] \mathbin{\&} \forall y(\varphi[y] \to y =_p x))$. But then one cannot claim any more that the "whole is the many counted as one". Also the question why counting disjuncts and not, e.g., maternally identicals remains.

□

Some formulations of the assumptions and theorems in the language of *Prover9/Mace4*:

```
%%%%%%%%%%%%%%%%%%%%%%%%%
%AP0
(all x(all y((all z(E(z,x)<->E(z,y)))->(all z(E(x,z)<->E(y,z)))))).
%AP2
((all x(exists y(E(y,x))))&(all x(all y(E(x,y)<->(((exists z(E(z,x))&(all z(E(z,x)->E(z,y))))&(all z1(all z2((E(z1,x)&E(z2,x)->E(z1,z2)))))))))).
%AP3
((all x(exists y(E(y,x))))&(all x(all y(E(x,y)<->((exists z(E(z,x)))&(all z(E(z,x)->E(z,y)))))))).
%AP4
((all x(exists y(E(y,x))))&(all x(all y(E(x,y)<->(exists z(E(z,x)&E(z,y))))))).
%%%%%%%%%%%%%%%%%%%%%%%%%
%DI1
(all x(all y(I(x,y)<->(all z(E(z,x)<->E(z,y)))))).
%DM1: M for improper part
(all x(all y(M(x,y)<->(all z(E(z,x)->E(z,y)))))).
%DM2: P for proper part
(all x(all y(P(x,y)<->(M(x,y)&-I(x,y))))).
%DM3: O for overlapping
(all x(all y(O(x,y)<->(exists z(E(z,x)&E(z,y)))))).
```

```
%DM4: A for atom
(all x(A(x)<->-(exists y(E(y,x)&-I(y,x))))).
%DM5: U for universal
(all x(U(x)<->(-A(x)&(all y(M(x,y)->x=y))))).
%[421] AM1: S for sum (existence and uniqueness are built in)
((all x(all y(all z(S(x,y,z)<->(all z1(O(z1,z)<->(O(z1,x)—O(z1,y))))))))&(all x(all y(exists z(S(x,y,z)&(all z1(S(x,y,z1)->I(z1,z)))))))).
%%%%%%%%%%%%%%%%%%%%%%%%
%OII
(all x(all y(S(x,y,z)<->((E(x,z)&E(y,z))&(all z1(E(z1,z)->(E(z1,x)—E(z1,y))))))))
%OIC
(exists x(exists y(exists z(((((((-I(x,y)&-I(x,z))&-I(y,z))&A(x))&A(y))&A(z))&(all z1(A(z1)->((I(z1,x)—I(z1,y))—I(z1,z)))))))))
%%%%%%%%%%%%%%%%%%%%%%%%
%T5
(all x(E(x,x)))
%T6
(all x(all y(all z((E(x,y)&E(y,z))->E(x,z)))))
%T7
(all x(I(x,x)))
%T8
(all x(all y(I(x,y)->I(y,x))))
%T9
(all x(all y(all z((I(x,y)&I(y,z))->I(x,z)))))
%T11
(all x(all y((all z(E(x,z)<->E(y,z)))->I(x,y))))
%T12
(all x(all y(I(x,y)<->(all z(E(x,z)<->E(y,z))))))
%T13
(all x(all y(I(x,y)<->(E(x,y)&E(y,x)))))
%T14
(all x(all y((I(x,y)&(exists z1(exists z2(exists z3(((((((-I(z1,z2)&-I(z1,z3))&-I(z2,z3))&E(z1,x))&E(z2,x))&E(z3,x))&(all z4(E(z4,x)->((I(z4,z1)—I(z4,z2))—I(z4,z3))))))))))->(exists z1(exists z2(exists z3(((((((-I(z1,z2)&-I(z1,z3))&-I(z2,z3))&E(z1,y))&E(z2,y))&E(z3,y))&(all z4(E(z4,y)-
```

>((I(z4,z1)—I(z4,z2))—I(z4,z3)))))))))))

%[422] T15

(all x(all y((exists z(E(z,x)&E(z,y)))<->(exists z(M(z,x)&M(z,y))))))

%T16

(all x(-(exists y(E(y,x)&-I(x,y)))<->-(exists y(P(y,x)))))

%T17

(all x(-P(x,x)))

%T18

(all x(all y(P(x,y)->-P(y,x))))

%T19

(all x(all y(all z((P(x,y)&P(y,z))->P(x,z)))))

%T20

(all x(all y(E(x,y)->M(x,y))))

%T21

(all x(all y(M(x,y)->E(x,y))))

%T22

(all x(all y(exists z(all z1(O(z1,z)<->(O(z1,x)—O(z1,y)))))))

%T23

(all x(all y(all z(S(x,y,z)<->S(y,x,z)))))

%T24

(all x(all y(all z(all z1(all z2(all z3(((S(x,y,z1)&S(z1,z,z2))&S(y,z,z3))-
>S(x,z3,z2))))))))

%T25

(all x(S(x,x,x)))

%T31

(all x(all y(M(x,y)<->(all z(O(z,x)->O(z,y))))))

%[423] T32

(all x(all y(I(x,y)->(all z(O(z,x)<->O(z,y))))))

%T33

(all x(all y(((A(x)&A(y))&-I(x,y))->-O(x,y))))

%%%%%%%%%%%%%%%%%%%%%%%%

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