The Synchronized Aggregation of Beliefs and Probabilities

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Project Information

Talk(s):

- Feldbacher-Escamilla, Christian J. (2017-09-24/2017-08-27). *Stability Preservation in Social Context*. Conference. Presentation (contributed). XXIV. Kongress der Deutschen Gesellschaft für Philosophie: Norm und Natur. HU Berlin: German Society for Philosophy (DGPhil).
- Feldbacher-Escamilla, Christian J. and Thorn, Paul D. (2016-07-04/2016-07-06). *The Synchronized Aggregation of Beliefs and Probabilities*. Conference. Presentation (contributed). European Epistemology Network (EEN) 2016. EHESS: Institut Jean-Nicod, CNRS, Paris IV.
- Feldbacher-Escamilla, Christian J. (2016-03-08/2016-03-11). *The Synchronized Aggregation of Beliefs and Probabilities*. Conference. Presentation (contributed). GWP.2016. University of Duesseldorf: GWP & DCLPS.

Introduction

Classical epistemology and philosophy of science: individual belief, degrees of belief, justification, knowledge, . . .

Social epistemology and modern approaches in the *pos*: consideration also of collective/group agency;

Relevant topics:

- bridging degrees of belief and belief (also: ‘binarization’)
- bridging individual beliefs/degrees of belief and collective ones

Here we consider bridging degrees of belief and belief in a collective setting: Are they synchronized?

Contents

BB: Belief Binarization

Intro

Two important notions: *Bel* and *Pr*

(Bel)
$Bel(\top),\ \neg Bel(\bot),$
$Bel(\varphi)$ and $\varphi \vdash \psi \;\Rightarrow\; Bel(\psi),$
$Bel(\varphi)\ \&\ Bel(\psi) \;\Rightarrow\; Bel(\varphi\ \&\ \psi)$

(Pr)
$Pr(\top) = 1,\ Pr(\varphi) \geq 0,$
$\varphi \lightning \psi \;\Rightarrow\; Pr(\varphi \vee \psi) = Pr(\varphi) + Pr(\psi),$
$Pr(\psi) > 0 \;\Rightarrow\; Pr(\varphi|\psi) = \frac{Pr(\varphi\ \&\ \psi)}{Pr(\psi)}$

Lockean Bridging:

$$\textbf{(L)}\ Bel(\varphi) \;\Leftrightarrow\; Pr(\varphi) \geq r \geq \tfrac{1}{2}$$

E.g.: You believed 'Hillary Clinton will be ...', because ...there was no alternative ...

The Lottery Paradox

Henry Kyburg's well-known example (1961):

Assume:

- $Pr(t_1 = w) = \cdots = Pr(t_{1.000.000} = w) = \frac{1}{1.000.000}$
- $Pr(t_1 = w) + \cdots + Pr(t_{1.000.000} = w) = 1$

Then, by help of (Bel), (Pr), (L) we get:

- We get $Bel(t_1 = w \vee \cdots \vee t_{1.000.000} = w)$
- But also $Bel(t_1 \neq w)$ & ... & $Bel(t_{1.000.000} \neq w)$
- And by &-closure: $Bel(t_1 \neq w$ & ... & $t_{1.000.000} \neq w)$
- Hence, again by &-closure: $Bel(\bot)$, hence ↯

So, at least at first glance, (Bel), (Pr), (L) seem to be incompatible.

STB: The Stability Theory of Belief

Hannes Leitgeb's stability approach (2014):

Two principles:

1. Re-interpretation of the scopes of the hidden quantifiers in (L): Instead of $\exists r \forall Pr$(L) assume $\forall Pr \exists r$(L)
2. Fit r (relevantly < 1) to your set of beliefs by a stability constraint:

$$\varphi \text{ is } Pr\text{-stable-}r \text{ iff for all } \psi\text{: } \varphi \not\perp \psi \Rightarrow Pr(\varphi|\psi) \geq r$$

Leitgeb's adequacy-result: The representation theorem:

Theorem (cf. Leitgeb 2014, p.140)

(Bel), (Pr), (L) iff (Bel) is Pr-stable-r axiomatizable.

The STB-Solution to the Paradox

It explains our intuitions on

- 'Surely ticket t_i wont win.', and
- 'Surely some ticket will win.'

by reference to different contexts:

- Context: $\overset{①}{t_i \neq w}$ **vs.** $\overset{②}{t_1 = w \vee \cdots \vee t_{i-1} = w \vee t_{i+1} = w \vee t_{1.000.000} = w}$
  Solution: *Pr*-stable axiomatizable is $Bel(②)$, but also $Bel(① \text{ \& } ②)$.

- Context: $t_1 = w$ **vs.** ... **vs.** $\overset{③}{t_i = w}$ **vs.** ... **vs.** $t_{1.000.000} = w$
  Solution: *Pr*-stable axiomatizable is only $Bel(③ \vee ②)$.

Some Problems of the STB-Solution

Main discussions on STB are about:

- the context-sensitivity of the choice of r
- the limited possibilites for Bel – for further impossibility results cf. (Rott)

Further Application of STB

Nevertheless, STB seems to bring about the right results also when applied to further specifications of (Bel) and (Pr).

Take, e.g., revision:

- for the domain of *Bel* we have principles of *belief revision*, the AGM postulates, connecting Bel_{new} with Bel_{old}
- for the domain of *Pr* we have principles of Bayesian update: conditionalization, connecting Pr_{new} with Pr_{old}

Here *Pr*-stability is preserved (cf. Leitgeb 2013);

But what about aggregation? Is *Pr*-stability also preserved among aggregations from individual beliefs/degrees of belief to collective ones?

JA: Judgement Aggregation

Intro

The problem of judgement aggregation:

	φ	ψ	χ
Bel_1/Pr_1	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$
Bel_2/Pr_2	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$
Bel_3/Pr_3	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$	$\{0,1\}/[0,1]$
$Bel_{\{1,2,3\}}/Pr_{\{1,2,3\}}$	?	?	?

Qualitatively: $Bel_{\{1,2,3\}} = aggr(Bel_1, Bel_2, Bel_3)$

Quantitatively: $Pr_{\{1,2,3\}} = aggr(Pr_1, Pr_2, Pr_3)$

Problem: Characteristics of *aggr*?

Minimal Conditions for JA I

Minimal requirements for aggregating beliefs and degrees of beliefs in groups of size n are (cf. List&Pettit 2002): $aggr : Bel^n/Pr^n \longrightarrow Bel/Pr$ with:

- **(U)** Universality: *aggr* allows as input any *Bel*, *Pr* satisfying (Bel), (Pr).

- **(A)** Anonymity: *aggr* cannot identify any specific input

  $aggr(Bel_1, \ldots, Bel_n) = aggr(Bel_1, \ldots, Bel_n, Bel_{n-1}) = \ldots$; similarly for the aggregation of *Pr*;

Minimal Conditions for JA II

Furthermore, *aggr* is systematic (transparent):

- **(S)** Systematizity: *aggr* is functional an propositionwise

$Bel_{\{1,\ldots,n\}}(\varphi) = aggr^*(Bel_1(\varphi), \ldots, Bel_n(\varphi))$
where $aggr^*(Bel_1(\varphi), \ldots, Bel_n(\varphi)) = aggr(Bel_1, \ldots, Bel_n)(\varphi)$;
similarly for Pr;

Impossibility: Beliefs

Take, e.g.:

	φ	ψ	φ & ψ
Bel_1	1	1	1
Bel_2	1	0	0
Bel_3	0	1	0
$Bel_{\{1,2,3\}}$	1	1	0

Here an aggregation by majority voting ($Bel_{\{1,2,3\}}$) produces an incoherent result.

A general impossibility result:

Theorem (cf. List&Pettit 2002)

(Bel), (U), (A), (S) are not jointly satisfiable by any aggr.

Impossibility: Degrees of Belief

Considering a further constraint:

- **(IP)** Independence Preservation: *aggr* preserves probabilistic independences in groups

I.e.: If $Pr_i(\varphi|\psi) = Pr_i(\varphi)$ $(1 \leq i \leq n)$), then also $aggr(Pr_1, \ldots, Pr_n)(\varphi|\psi) = aggr(Pr_1, \ldots, Pr_n)(\varphi)$;

one also ends up with an impossibility result for degrees of belief:

Theorem (cf. Lehrer&Wagner 1983)

(Pr), (U), (A), (S), (IP) are not jointly satisfiable by any aggr.

Solutions

Solutions to these problems are:

- Vs. (U) by domain restriction (e.g. by ensuring convergence)
- Vs. (A) by favouring, e.g., expert judgements
- Vs. (S) by structuring the propositions before the aggregation (e.g. premise-based approach)
- Vs. (IP) by accepting different update behaviour
- Vs. the choice of a single *aggr* by a purpose dependent choice of different *aggr*s

One might ask whether BB, especially STB, provides some help in figuring out further solutions (e.g. vs. (U))?

But also, as questioned above: Is *Pr*-stability-*r* synchronizing $Bel_{\{1,\ldots,n\}}$ and $Pr_{\{1,\ldots,n\}}$?

BB+JA

Intro

A short upshot:

- BB: *Bel* and *Pr* can be bridged by *L*, if *Bel* is *Pr*-stable-*r* axiomatizable.
- JA: Some properties within a group cannot be preserved generally in collective judgements: e.g., (IP), given (Pr), (U), (A), (S);
- BB+JA: Is *Pr*-stability-*r* preserved in collective judgements?

Two Types of Stability in JA

In JA *Pr* may vary among the members of a group.

But also *r* might vary. Depending on variation we may distinguish two types of stability-preservation ($1 \leq i \leq n$):

- Global: Bel_i is Pr_i-stable-r axiomatizable.
- Local: Bel_i is Pr_i-stable-r_i axiomatizable.

Stability Preservation as a Desideratum in JA?

One might ask why universal properties of individual beliefs/degrees of belief should be preserved in pooling them?

A general answer might be seen in the *maximization* of individual interests and by this also the increased acceptability of a pooling result.

So, a general pooling-maxim might be: If each Bel_i or Pr_i has property Q, then also $aggr(Bel_1, \ldots, Bel_n)$ or $aggr(Pr_1, \ldots, Pr_n)$ should have Q.

E.g.: (IP); in case of comparability one might prefer that aggregation method that maximizes the preservation of universal properties.

Local Stability Preservation

The explicit formulation of the local stability preservation constraint is as follows:

- **(LSP)** Local Stability Preservation: If Bel_i can be Pr_i-stable-r_i axiomatized ($1 \leq i \leq n$; for some $r_1, \ldots, r_n < 1$), then also $aggr(Bel_1, \ldots, Bel_n)$ can be $aggr(Pr_1, \ldots, Pr_n)$-stable-r axiomatized (for some $r < 1$).

One can observe that:

Observation

(Bel), (Pr), (LSP) is not generally satisfied by $aggr$.

Global Stability Preservation

The explicit formulation of the global stability preservation constraint is as follows:

- **(GSP)** Global Stability Preservation: If there is a unique Pr_i-stable-r axiomatization of Bel_i $(1 \leq i \leq n)$, then also $aggr(Bel_1, \ldots, Bel_n)$ can be $aggr(Pr_1, \ldots, Pr_n)$-stable-r axiomatized.

One can observe that:

Observation

(Bel), (Pr), (GSP) is satisfied by any linear $aggr$.

(where such a method can always be described by $aggr(Pr_1, \ldots, Pr_n)(\varphi) = \sum_{1 \leq i \leq n} w_i \cdot Pr_i(\varphi)$)

Summary

- One candidate for belief binarization or bridging: STB
- STB has some faults, but seems to be quite natural inasmuch as stability is preserved among classical solutions for the different domains
- E.g.: Belief revision and Conditionalisation
- This continues also in the social setting: GSP

References I

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Appendix

Ad (GSP):

- Assume Pr_i-stability-r amongst the group.
- Then, there is a φ such that for any ψ: $Pr_i(\varphi|\psi) \geq r$.
- Since linear opinion pooling is convex, we get $aggr(Pr_1, \ldots, Pr_n)(\varphi|\psi) \geq r$.
- Hence, φ is also $aggr(Pr_1, \ldots, Pr_n)$-stable-r.