Suppositional Reasoning

Its Logic and Causal Structure

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Project Information

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Introduction

Suppositions come in different forms and moods.

Typically, suppositional reasoning is dealt within the framework of probabilistic updating.

In this talk, we want to link this discussion better to that of causal reasoning.

Contents

Suppositional Reasoning as Probabilistic Updating

Suppositional Reasoning

Suppositions come in two different basic moods:

Indicative Mood: Supposing the truth of A amounts to revising one's epistemic state in exactly the way as if one learns the truth of A.

Subjunctive Mood: Supposing the truth of A amounts to revising one's epistemic state in exactly the way as if one learns that A had been made true by some "local miracle".

Furthermore, we distinguish between full and partial suppositions:

Full: treating A as certain knowledge

Partial: treating A as having an increased degree of plausibility

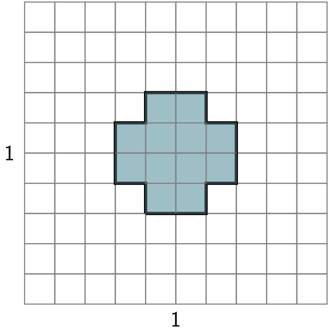
Combine these categories (cf. Eva and Hartmann 2021):

	full	**partial**
indicative	Full Indicative Supposition	Partial Indicative Supposition
subjunctive	Full Subjunctive Supposition	Partial Subjunctive Supposition

Probabilistic Updating: Visualisation/Framework

Finite set of possible worlds $W = \{w_1, w_2, \ldots, w_n\}$

Each possible world has a probability Pr: $Pr(w_i) \geq 0$ and $\sum_{i=1}^{n} Pr(w_i) = 1$

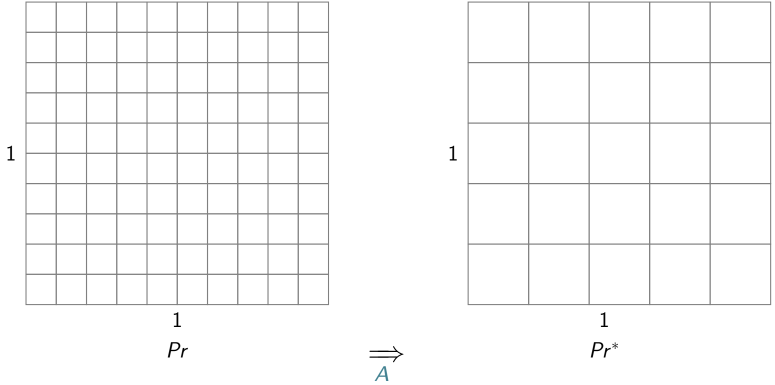


Proposition: set of possible worlds in which it is true

Probability of a proposition: sum of the probabilities of its possible worlds

Probabilistic Updating: Visualisation/Framework

Updating on a proposition A brings in some dynamics:



Probabilistic Updating: Conditionalization

If we learn that a proposition A is true, it is rational to apply Bayesian updating or conditionalization.

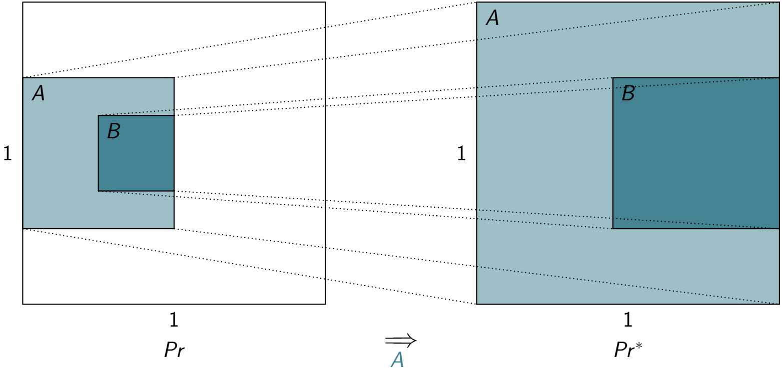
The idea is that we narrow down our "universe" to just the part or those possible worlds where A is true.

The new probability (Pr^*) of any proposition will be simply the (old) probability of that proposition "in the light of" A:

$$Pr^*(\cdot) = Pr(\cdot|A)$$

E.g., to learn A implies to incorporate it with certainty: $Pr^*(A) = 1$ (this is simply the result of conditionalization itself: $Pr(A|A) = 1$)

Probabilistic Updating: Conditionalization



Since conditionalization is about taking A as certain, it is a form of full supposition in the indicative mood:

Full Indicative Supposition	Partial Indicative Supposition
Full Subjunctive Supposition	Partial Subjunctive Supposition

Probabilistic Updating: Jeffrey Conditionalization

Now, what if we are not fully certain about A?

Jeffrey (1983) proposed a generalisation for this case.

The main idea is that what should be held fixed in updating are the conditional probabilities, they are rigid:

$$Pr^*(X|Y) = Pr(X|Y)$$

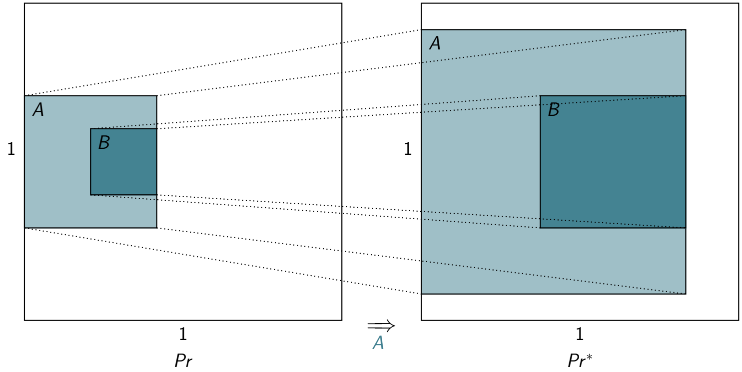
This assumption allows us do calculate the updated probabilities:

1. $Pr^*(X|Y) = Pr(X|Y)$ and $Pr^*(X|\overline{Y}) = Pr(X|\overline{Y})$ (by rigidity)
2. $Pr^*(X,Y) = Pr(X|Y) \cdot Pr^*(Y)$ and $Pr^*(X,\overline{Y}) = Pr(X|\overline{Y}) \cdot Pr^*(\overline{Y})$
3. $Pr^*(X) = Pr(X|Y) \cdot Pr^*(Y) + Pr(X|\overline{Y}) \cdot Pr^*(\overline{Y})$ (by adding the left and the right terms)

Hence, a generalised form of updating on uncertain or certain A in the indicative mood is Jeffrey Conditionalization:

$$Pr^*(\cdot) = Pr(\cdot|A) \cdot Pr^*(A) + Pr(\cdot|\overline{A}) \cdot Pr^*(\overline{A})$$

Probabilistic Updating: Jeffrey Conditionalization



Since Jeffrey conditionalization is about taking A as only more plausible, it is a form of partial supposition in the indicative mood:

Full Indicative Supposition	Partial Indicative Supposition
Full Subjunctive Supposition	Partial Subjunctive Supposition

Probabilistic Updating: Imaging

What happens if we do not only want to learn about something being more certain, but also that something had been made more certain?

Lewis (1976) suggested an influential update rule for this that he labelled "imaging". (For explanatory purposes, we will stick to the initial and quite restricted form of imaging.)

The update rule is more fine-grained than conditionalization inasmuch as it relies on features of possible worlds underlying our propositions.

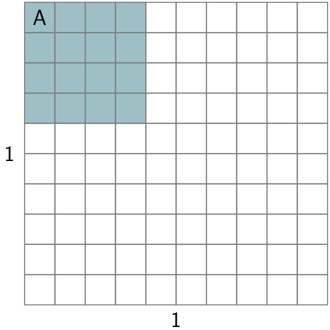
One feature of possible worlds we assume for basic imaging is that each possible world w has a, with respect to the truth of a proposition A, closest or most similar world w'.

E.g., if A is true in w, then w is most similar to w w.r.t. A.

If A is false in w, we look for that w' that minimally revises w for A's truth.

Probabilistic Updating: Imaging

For our universe-grid, this means that we map each possible world w.r.t. a proposition A to another possible world.



Now, the main idea of updating by imaging is that the image on A of a probability function can be computed by shifting the original probability of each world w over to its closest world w' in which A is true.

Probabilistic Updating: Imaging

Technically, we can define the rule by the help of a transition function T_A.
We suppose for such a function: for all w there is exactly one w' (the most similar w'): $T_A(w', w) = 1$; all other w'' are assigned 0: $T_A(w'', w) = 0$
We can then define the image of Pr on A as:

$$Pr(\{w_i\}|_T A) = \sum_{w \in W} Pr(\{w\}) \cdot T_A(w, w_i)$$

And for propositions:

$$Pr(X|_T A) = \sum_{w \in X} Pr(\{w\}|_T A)$$

The update rule based on learning that A was brought about is simply:

$$Pr^*(\cdot) = Pr(\cdot|_T A)$$

Since $Pr^*(A) = 1$, it is a form of full supposition in the subjunctive mood:

Full Indicative Supposition	Partial Indicative Supposition
Full Subjunctive Supposition	Partial Subjunctive Supposition

Probabilistic Updating: Imaging

Let us illustrate this by the help of an example (taken from Leitgeb 2016):

w_1 H_1: fruit basket contains exactly 1 apple + 1 banana

w_2 H_2: fruit basket contains exactly 1 pear

w_3 H_3: fruit basket contains exactly 1 apple

Evidence A: The banana was removed if it had been there at all.

Similarity/Closeness:

- $T_A(w_1, w_1) = 0$
- $T_A(w_1, w_2) = 0$
- $T_A(w_1, w_3) = 1$
- $T_A(w_2, w_1) = 0$
- $T_A(w_2, w_2) = 1$
- $T_A(w_2, w_3) = 0$
- $T_A(w_3, w_1) = 0$
- $T_A(w_3, w_2) = 0$
- $T_A(w_3, w_3) = 1$

Initial (Pr):

- $Pr(H_1) = Pr(H_2) = Pr(H_3) = 1/3$

Updated (Pr^*):

- $Pr(H_1|_T A) = Pr(w_1) \cdot 0 + Pr(w_2) \cdot 0 + Pr(w_3) \cdot 0 = 0$
- $Pr(H_2|_T A) = Pr(w_1) \cdot 0 + Pr(w_2) \cdot 1 + Pr(w_3) \cdot 0 = 1/3$
- $Pr(H_3|_T A) = Pr(w_1) \cdot 1 + Pr(w_2) \cdot 0 + Pr(w_3) \cdot 1 = 2/3$

Probabilistic Updating: Jeffrey Imaging

Now, what if we are not fully certain about A?

Eva and Hartmann (2021) suggest a generalisation "formally" similar to that of Jeffrey for Bayesian conditionalisation:

The idea is simply to not fully "stretch out" A towards the borders of the grid, but only partially:

$$Pr^*(\cdot) = Pr(\cdot|_T A) \cdot Pr^*(A) + Pr(\cdot|_T \overline{A}) \cdot Pr^*(\overline{A})$$

Since $Pr^*(A)$ is only taken to be more plausible, it is a form of partial supposition in the subjunctive mood:

Full Indicative Supposition	Partial Indicative Supposition
Full Subjunctive Supposition	Partial Subjunctive Supposition

Suppositional Reasoning as Probabilistic Updating

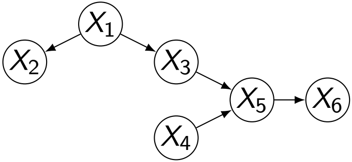
	full	**partial**
indicative	Conditionalization	Jeffrey Conditionalization
subjunctive	Imaging	Jeffrey Imaging

Suppositional Reasoning as Causal Reasoning

Bayesian Networks (BNs)

A Bayesian network is a triple $\langle \mathbf{V}, \mathbf{E}, P \rangle$, such that ...

- $\mathbf{V}$ is a set of variables $X_1, \ldots, X_n$.
- $\mathbf{E}$ is a binary relation on $\mathbf{V}$ ($X_i \longrightarrow X_j$).
- P is a probability distribution over $\mathbf{V}$.



$\mathbf{Par}(X_i)$ ... the set of X_i's parents

$\mathbf{Des}(X_i)$ ... the set of X_i's descendants

Definition (Markov condition)

$\langle \mathbf{V}, \mathbf{E}, P \rangle$ satisfies the Markov condition iff every $X \in \mathbf{V}$ is probabilistically independent of its non-descendants conditional on its parents (Pearl 2000):

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | \mathbf{Par}(X_i))$$

An Example

Alice works in an office. A new fire alarm system monitoring the whole building has recently been installed.

Assume that Alice supposes that the alarm goes off.

If she does so in an indicative mood, then she will also come to a couple of other beliefs:

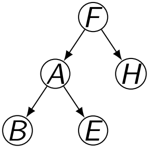
- that there might be a *F*ire F
- that the building soon will get very *H*ot H
- that the alarm was also heard by her colleague *B*ob B
- that the building will be *E*vacuated E

If Alice supposes the alarm to go off in a subjunctive mood, on the other hand, then she will only come to believe:

- *B*ob
- *E*vacuation

She would only have to come to the beliefs regarding the effects of the alarm going off, but not about the event's possible cause.

An Example



A: whether the alarm goes off
F: whether there is fire
H: whether it is hot in the building
B: whether Bob hears the alarm
E: whether the building is evacuated

Full Indicative Suppositions

This is the easiest case to cover. It can be captured by ordinary conditionalization on the basis of the causal structure:

$$Pr^*_a(\cdot) = Pr(\cdot|a)$$

Since F, H, B, E are each d-connected (i.e. connected without a collider) to A, full indicative supposition of a might lead to a change in degrees of belief in each of the events represented by these variables.

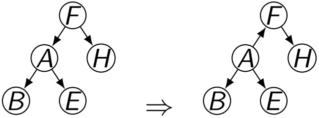
This fits our intuitive treatment of the exemplary case above: If Alice supposes a in the indicative mood, i.e., treats a as certain knowledge, then her degrees of belief of f, h, b, and e can be expected to go up as well.

Partial Indicative Suppositions

This case requires that we increase a's probability to a degree less than 1.

But if A is not exogenous, which is the case in our causal structure in underlying the Alice example, we cannot simply change A's distribution without changing some of the BN's other parameters at the same time.

However, there is a technical trick to calculate A's impact: We can construct a probabilistically equivalent BN in which A is exogenous:



(Problem:) In the second graph, the arrows can no longer be interpreted as causal arrows.

But we can change A's probability distribution without changing any of the BN's other parameters.

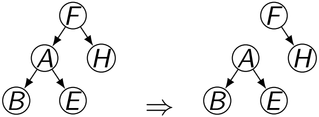
This allows us to compute the epistemic effects of Alice supposing a in the part. ind. mood.

As in the case of full ind. supposition, by d-connection we get some but less impact.

Full Subjunctive Suppositions

We propose to handle full subjunctive suppositional reasoning in terms of *surgical interventions* (cf. Pearl 2000).

The idea is to first delete all arrows pointing at A:



Next, one assumes that A takes value a.

Pearl: Deleting all arrows incoming to A and assigning probability 1 to the event that A takes on a is marked with a hat symbol $\hat{a}$; the post-intervention distribution is $Pr_{\hat{a}}(\cdot) = Pr(\cdot|\hat{a})$

So we can compute Alice's updated distribution after full subj. supp. of a:

$$Pr^*_a(\cdot) = Pr_{\hat{a}}(\cdot)$$

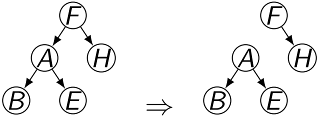
Partial Subjunctive Suppositions

To partially subjunctively suppose *a* faces the same problem as we had w.r.t. the indicative mood: A is not exogenous.

However, we do not need to apply a "technical trick" (losing grip of the causal structure) here.

The reason is simply that if we intervene on A, then A becomes exogenous.

So, first we calculate the post-intervention distribution based on:



Next, we increase a's probability to a value less than 1. We designate this probability with $Pr_{\hat{A}}(\cdot)$. Updating in the partial subjunctive mood results in:

$$Pr^*_a(\cdot) = Pr_{\hat{A}}(\cdot)$$

Suppositional Reasoning as Causal Reasoning

	full	**partial**
indicative	Conditionalization $Pr(\cdot \vert a)$	Single Value-Manipulation $Pr_A(\cdot)$
subjunctive	Intervention $Pr_{\hat{a}}(\cdot)$	Generalized Intervention $Pr_{\hat{A}}(\cdot)$

Probabilistic Updating and Causal Reasoning

Adequacy (WIP)

Does our treatment of suppositions in the causal setup coincide with the different update rules?

	full	**partial**
indicative	✓	✓
subjunctive	✓ Probably	wip

References I

Eva, Benjamin and Hartmann, Stephan (2021-01). "The Logic of Partial Supposition". In: *Analysis* 81.2, pp. 215–224. DOI: 10.1093/analys/anaa060.

Gebharter, Alexander and Feldbacher-Escamilla, Christian J. (submitted). "A Causal Theory of Supposition". In: *manuscript*.

Jeffrey, Richard C. (1983). *The Logic of Decision*. Second Edition. Chicago: The University of Chicago Press.

Leitgeb, Hannes (2016). "Imaging All the People". In: *Episteme*, pp. 1–17. DOI: 10.1017/epi.2016.14.

Lewis, David (1976). "Probabilities of Conditionals and Conditional Probabilities". In: *The Philosophical Review* 85.3, pp. 297–315. DOI: 10.2307/2184045.

Pearl, Judea (2000). *Causality. Models, Reasoning, and Inference*. Cambridge: Cambridge University Press.