On the Inductive Validity of Conclusions by Analogies

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Introduction

Analogical thinking is important in hypothesis invention, but also in reasoning.

In order to justify analogical reasoning, the research programme of logical probability was modified quite a lot.

In this talk we are going to reconstruct some of the modifications. We will try to pose some of their main problems and provide an evaluation within the methodology of scientific research programmes.

Contents

Evaluation Criteria

Scientific Research Programmes

Conceptual background for our evaluation:

- One goal of philosophy of science is the reconstruction of scientific theories.
- For this purpose the expression 'scientific research programme', introduced by Imre Lakatos (Lakatos 1980), seems to be very fruitful.
- Without taking care of the important methodological part (i.e. a set of normative statements) it holds that:

Definition

T is a scientific research programme iff there are C, $H_1, \ldots, H_n$ and $T_1, \ldots, T_n$ such that:

- $T_1, \ldots, T_n$ are theories, and:
- $T_1 = Cn(C \cup H_1)$ and $\ldots$ and $T_n = Cn(C \cup H_n)$, and:
- $T = \langle T_1, \ldots, T_n \rangle$.

Degenerative Research Programmes

Conceptual background for our evaluation – continued:

- "Let us call a problem shift progressive if it is both theoretically [i.e. each new theory has has some excess empirical content over its predecessor] and empirically progressive [i.e. some of the excess empirical content is also corroborated], and degenerating if it is not." (cf. Lakatos 1980, p.34)
- That is: "Progressive problem shifts are those that lead to genuine predictions; degenerative ones are those that lead to false predictions or offer mere accommodation." (Forster and Shapiro 2000, p.33)
- Since we'll not consider an empirical theory, but the theory of confirmation, we will concentrate here on the accommodation-part and will interpret it more generally as some kind of (over-)fitting via increased parametrization.
- So, a research programme is degenerative, e.g., if its number of parameters increases while the problem content remains unchanged.

Degenerative Research Programmes

An Example:

x	y	$y = x^2$	$y = x^3 - \ldots$
1	1	1	1
2	4	4	4
3	9	9	9
4	20	16	20

Conclusions by Analogy

Conclusions by Analogy: An Example

Judith J. Thomson's argumentation in favour of abortion:

> *"Imagine this. You wake up in the morning and find yourself back to back in bed with an unconscious violinist. A famous unconscious violinist. He has been found to have a fatal kidney aliment, and so the Society of Music Lovers has [. . .] kidnapped you, and last night the violinist's circulatory system was plugged into yours, so that your kidneys can be used to extract poisons from his blood as well as your own. The director of the hospital now tells you, 'Look, we're sorry the Society of Music Lovers did this to you – we would never have permitted it if we had known. But still, they did it, and the violinist now is plugged into you. To unplug you would be to kill him. But never mind, it is only for nine months. By then he will have recovered from his aliment, and can safely be unplugged from you'." (cf. Thomson 1971, pp.48f)*

This case's similarity relation:

$$\begin{array}{lcr}
\text{Foetus} & \cong & \text{Violinist} \\
\text{Mother} & \cong & \text{Abductee} \\
\text{Getting unintendetly pregnant} & \cong & \text{Kidnapping} \\
\text{Pregnancy} & \cong & \text{Dialysis treatment} \\
\text{Abortion} & \cong & \text{Unplugging}
\end{array}$$

Conclusions by Analogy: Characterization

A very general scheme:

- Individuals c_1 and c_2 share some relevant properties.
- They may also fail in sharing some properties, but not the relevant ones.
- Now you learn that c_1 has property P_1.
- And you conclude that also c_2 has property P_1.

Consider the example of Thomson:

- The mother and the abductee as well as the foetus and the violinist share some relevant properties.
- They also fail in sharing some properties, but not that relevant ones (gender, age, etc. doesn't seem to matter very much).
- Now you learn that in the violinist case unplugging is ethically justified.
- And you conclude that also abortion is ethically justified.

The Task: Similar to the Problem of Induction

E.g.: How to justify enumerative induction?

1. $P_1(c_1)$
2. $\vdots$
3. $P_1(c_n)$
4. Hence: $P_1(c_{n+1})$

There is of course no deductive justification: $P_1(c_1) \& \ldots \& P_1(c_n) \nvdash P_1(c_{n+1})$.

But what about an inductive one?

Carnap's Programme of Logical Probability

In order to justify, e.g., enumerative induction, Rudolf Carnap "initiated" the programme of logical probability (cf. Carnap 1950/1962, p.202):

> *"We shall see that a statement of deductive logic like 'e L-implies h' means the entire range of e is included in that of h, while a statement of inductive logic like '$\mathfrak{c}(h, e) = 3/4$' means three-fourths of the range of e is included in that of h."*

E.g.: Monadic first-order language $\mathcal{L}^{m,n}$ with m individual constants and n monadic predicates. E.g.: $m = 2$ and $n = 1$ (cf. Carnap 1950/1962, pp.106f):

	$P_1(c_1)$	$P_1(c_2)$	predicate expressions
1	0	0	$\sim P_1(c_1) \& \sim P_1(c_2)$
2	0	1	$\sim P_1(c_1) \& P_1(c_2)$
3	1	0	$P_1(c_1) \& \sim P_1(c_2)$
4	1	1	$P_1(c_1) \& P_1(c_2)$

A probability assignment over a language $\mathcal{L}^{m,n}$ is an assignment of $\in [0, 1]$ to a formula's predicate expressions, such that they sum up to 1.

Carnap's Programme of Logical Probability

Now let us come to the definition of logical probability:

$$\mathfrak{c}(\varphi) = \frac{1}{2^{m \cdot n}}$$

... provided φ is a predicate expression. Otherwise it is the sum of the logical probabilities of its disjunction of predicate expressions.

Note: Every quantifier-free formula is equivalent to some disjunction of predicate expressions. So the method above works generally for all quantifier-free statements.

Let us consider analogical reasoning in the light of logical probability:

$$\mathfrak{c}(P_1(c_1)|P_1(c_2)) = \frac{\mathfrak{c}(P_1(c_1) \& P_1(c_2))}{P_1(c_2)} =$$
$$\frac{\mathfrak{c}(P_1(c_1) \& P_1(c_2))}{\mathfrak{c}(P_1(c_1) \& P_1(c_2)) + \mathfrak{c}(P_1(c_1) \& \sim P_1(c_2))} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

So $\mathfrak{c}$ does not allow for "learning from experience".

Carnap's Modification I

For this reason Carnap introduced a new parameter, λ, which should be an inverse measure for the speed of learning from experience (cf. Carnap 1950/1962, par.9):

$$\mathfrak{c}^{\lambda}(P_i(c_1)|\varphi) = \frac{n_i + \frac{\lambda}{\kappa}}{n+\lambda}$$

... where:

- n_i is the number of individual constants a, such that $\varphi \vDash P_i(a)$
- n is the number of all individual constants in φ
- κ is the number of possible predicate expressions of $\mathcal{L}^{m,n}$ ($\kappa = 2^n$).

In accordance with the fact that $\mathfrak{c}$ does not allow for learning from experience, it turns out that $\mathfrak{c} = \mathfrak{c}^{\lambda=\infty}$.

The range of $\mathfrak{c}^{\lambda}$ with $\lambda \in [0, \infty)$ is the so-called "*continuum of inductive methods*"

On the Validity of Perfect Analogies

Now, it also turns out that if λ is finite, then it holds that:

$$\mathfrak{c}^{\lambda}(P_1(c_1)|P_1(c_2)) > \frac{1}{2}$$

So, there is some learning from experience going on in the continuum.

More generally: The inference of enumerative induction, sometimes called also the "*inference via perfect analogies*", is valid in such λ-systems:

$$\mathfrak{c}^{\lambda}(P_1(c_{n+1})|P_1(c_2)\&\ldots\&P_1(c_n)) > \frac{1}{2}$$

But what about imperfect analogies? I.e. analogies with at least some dissimilarities?

On the Invalidity of Imperfect Analogies

It can be shown that for any λ: The $\mathfrak{c}^\lambda$-system allow for no justification of imperfect analogies.

For no $\mathfrak{c}^\lambda$-system it holds in general, e.g.:

$$\underset{\text{perfect analogy}}{\underset{\uparrow}{\mathfrak{c}^\lambda(P_1(c_1)|P_1(c_2))}} \;>\; \underset{\text{imperfect analogy}}{\underset{\uparrow}{\mathfrak{c}^\lambda(P_1(c_1)|P_1(c_2)\&P_2(c_1)\& \sim P_2(c_2))}} \;>\; \underset{\text{confirmation without learning}}{\underset{\uparrow}{\mathfrak{c}^\lambda(P_1(c_1))}}$$

For this reason Carnap introduced another parameter, γ, in order to cope with imperfect analogies (1963 und: (cf. Carnap 1959, app.B)).

Carnap's Modification II

The set of predicates of $\mathcal{L}^{m,n}$ is partitioned into families $\mathcal{F}_i$ (with $1 \leq i \leq n$) of different modalities. The elements of $\mathcal{F}_i$ are the most specific properties of $\mathcal{F}_i$

Example:

- $\mathcal{F}_{colour} = \{red, orange, yellow, green, \dots\}$
- $\mathcal{F}_{sound} = \{very\ loud, loud, \dots\}$

$$\vdots$$

(Of course in an application the $\mathcal{F}_i$'s are much more finely grained...)

One can interpret this parameter γ relativized to $P_i \in \mathcal{F}_i$ (for short: γ_i) as the prior probability that one of the m individuals has the property P_i.

Example: If $\mathcal{F}_2$ is the modality of sound and if there are, e.g., only four distinguishable sounds with equal extension in the sound space, then $\gamma_2 = \frac{1}{4}$

Carnap's Modification II

The λ-systems are modified to γ-λ-systems by:

$$\mathfrak{c}^\lambda(P_i(c_1)|\varphi) = \frac{n_i+\lambda\cdot\gamma_i}{n+\lambda}$$

In this definition γ_i is a logical factor (depends on our set up language) and n_i, n are empirical factors (contain information about the evidence we learn by)

What about λ? Since it is multiplied with the logical factor γ_1, increasing λ will put more weight on the logical factor. And that is to decrease the influence of the empirical factor, the evidence, the speed of learning – or in the analogy-interpretation: To decrease the influence of analogy in reasoning. Nice, isn't it?

But regarding imperfect analogies the γ-λ-systems have to be shown to work for languages with two predicates only (cf. Maher 2001, p.183).

So, where do we stand after γ-λ-parametrization? At monadic first-order logic with not more than two predicates and without quantification…

Evaluation & Summary

Is the Programme of Logical Probability Degenerative?

It seems so!

Just consider the problem content of the research programme: it remained quite unchanged for about fifty years now, regarding technicalities it was even narrowed down to monadic languages.

And the sequence of $\langle \mathfrak{c}, \mathfrak{c}^{\lambda}, \mathfrak{c}^{\gamma,\lambda}, \ldots, \mathfrak{c}^{\mu,\gamma,\lambda}, \ldots \rangle$ (for the latter cf. Niiniluoto 1981) seems to be more accommodation than producing excess content.

So, Lakatos' constraints for a research programme to be progressive are not satisfied.

But of course, even if the programme of logical probability and its aimed justification for analogical reasoning are degenerative, no one should be prevented from going on to try to fix the problems, or, as Lakatos put it:

> *"One may rationally stick to a degenerating research programme until it is overtaken by a rival and even after. What one must not do is to deny its poor public record. Both Feyerabend and Kuhn conflate methodological appraisal of a programme with firm heuristic advice about what to do. It is perfectly rational to play a risky game: what is irrational is to deceive oneself about the risk." (cf. Lakatos 1980, p.117)*

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