Lockean Thesis and Non-Probababilism

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Project Information

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Belief in All Its Varieties

Rational *belief*, a core notion of epistemology, comes in different forms:

- Qualitative: belief simpliciter (Hintikka 1962) $Bel(p)$
- Comparative: ranked belief (Spohn 2012) $\kappa(p) \leq \kappa(q)$
- Quantitative: degrees of belief (Ramsey 1926/1950) $Pr(p)$

As generally studied in measure theory, there are several connections. E.g.

- $\kappa(p) \leq \kappa(q)$ iff $Pr(p) \leq Pr(q)$

We are interested in the relation between *Bel* and *Pr* via a threshold rule:

- $Bel(p)$ iff $Pr(p) \geq r$

As is well known, such a rule comes with several paradoxes.

Contents

The Lottery Paradox

The Paradox

Consider a fair lottery with 1.000 tickets:

- $Pr(t_1 = t_l) = \cdots = Pr(t_{1,000} = t_l) = \frac{999}{1.000} = 0.999$
- $Pr(t_1 = t_l \,\&\, \cdots \,\&\, t_{1,000} = t_l) = 0.0$

By the threshold rule, i.e. the so-called Lockean Thesis:

- $Bel(p)$ iff $Pr(p) \geq r$

One gets for reasonable r:

- $Bel(t_1 = t_l) \,\&\, \cdots \,\&\, Bel(t_{1,000} = t_l)$
- $\neg Bel(t_1 = t_l \,\&\, \cdots \,\&\, t_{1,000} = t_l)$

Which runs against the rationality of *Bel*.

Principles of Belief Simpliciter

Ⓑ

B1 Consistency $Bel(\top)$ and $\neg Bel(\bot)$

B2 Single Premiss Closure $Bel(p),\ p \vdash q \ \Rightarrow\ Bel(q)$

B3 Conjunctive Closure/Conjunctivism $Bel(p),\ Bel(q) \ \Rightarrow\ Bel(p\&q)$

Principles of Degrees of Belief

Ⓟ

P1 Non-Negativity $Pr(p) \geq 0.0$

P2 Normalisation $Pr(\top) = 1.0$

P3 Finite Additivity $p \lightning q \Rightarrow Pr(p \vee q) = Pr(p) + Pr(q)$

The Bridging Principle

Ⓛ

$\exists r$:

L1 Leibniz Condition $r > 0.5$

L2 Fallibilism $r < 1.0$

$\forall p$:

L3 Threshold Rule $Bel(p)$ iff $Pr(p) \geq r$

Note the order of the quantifiers.

Also, more generally: $\forall Bel, Pr\ \exists r\ \forall p$

And not, e.g. $\exists r\ \forall Bel, Pr\ \forall p$

The Lottery Generator

The relevant part of the structure of the lottery is captured via:

(O)

$\forall r$:

O1 Leibniz Condition $r > 0.5$

O2 Fallibilism $r < 1.0$

$\exists p, q$:

O3 Structural Richness $Pr(p) \geq r,\ Pr(q) \geq r,\ Pr(p\&q) < r$

The idea is that for any threshold r a lottery case can be devised.

The Paradox Again

Henry Kyburg (Jr.) showed (cf. Kyburg (Jr.) 1961):

$$\text{Ⓑ} \;\&\; \text{Ⓟ} \;\&\; \text{Ⓛ} \;\&\; \text{Ⓞ} \;\vdash\; ↯$$

- $Pr(p) \geq r,\ Pr(q) \geq r,\ Pr(p\&q) < r$ (by Ⓞ)
- *Pr* satisfies the laws of degrees of belief (by Ⓟ)
- $Bel(p),\ Bel(q), \neg Bel(p\&q)$ (by Ⓛ)
- *Bel* does not satisfy the laws of belief simpliciter (by Ⓑ)

Classical Approaches

Revise Belief Simpliciter

Ⓟ & Ⓛ & Ⓞ $\vdash$ ¬Ⓑ

Richard C. Jeffrey opts for this position with his suggestion to eliminate the qualitative notion of rational belief completely from the epistemic realm (cf. Jeffrey 2004). This is vs. B1–B3 altogether.

Kyburg (Jr.) opted for this position, because he thought the importance of Ⓛ outweighs that of Ⓑ. For a critical discussion (cf. Schick 1963). For Ⓑ he distinguished different levels of rational inheritance and rational inference. The main idea is that $p\&q$ is on a different inference level as p and q are. This runs against conjunctivism, B3.

It seems plausible to assume that also paraconsistent logic is in this line of argumentation. From a *dialethic* point of view to belief in $\perp$ on the one side and to restrict inferences (non-explosion) on the other side seems to make sense. This is against B1,B2.

A Case for Conjunctive Closure: The Review Paradox

The most conservative revision of Ⓑ seems to be revising B3, conjunctivism.

However, such a revision also causes paradoxes. E.g. The Review Paradox:

Given:

- Lockean Bridging Ⓛ
- Bayesian Update Given evidence p appears, then $Pr_{t'}(q) = Pr_t(q|p)$
- Vacuous Belief Update If $Bel_t(p)$, evidence p, then $Bel_{t'}(q)$ iff $Bel_t(q)$
- Hence: Conjunctivism, i.e. B3.

(cf. Leitgeb 2014a)

A Case for Conjunctive Closure: The Review Paradox

To illustrate this, consider:

- Author $Bel_t(p),\ Bel_t(q),\ \neg Bel_t(p\&q)$
- Reviewer provides evidence p

Then:

1. $Pr_{t'}(q) = Pr_t(q|p) = \frac{Pr_t(p\&q)}{Pr_t(p)}$ (by Bayesian Update)
2. $Pr_{t'}(p\&q) = Pr_t(p\&q|p) = \frac{Pr_t(p\&q)}{Pr_t(p)}$ (by Bayesian Update)
3. Hence: $Pr_{t'}(q) = Pr_{t'}(p\&q)$
4. $Bel_{t'}(p),\ Bel_{t'}(q),\ \neg Bel_{t'}(p\&q)$ (by Vacuous Belief Update)
5. $Bel_{t'}(q)$ iff $Bel_{t'}(p\&q)$ (by 3 and Ⓛ)
6. Hence ↯

So, an author cannot take in/update on evidence provided by a reviewer.

Revise Bridging

(B) & (P) & (O) $\vdash$ ¬(L)

There are several ways of revising bridging:

- Apply no threshold rule (especially vs. L3).
  Problem: Standard procedure to move on from quantitative to qualitative notions.
- Give up the Leibniz condition—vs. L1.
  Problem: May result in inconsistent belief: $Pr(p) \geq r$, $Pr(\neg p) \geq r$ and by this $Bel(\bot)$
  Note that this is not necessarily the case. E.g. (Lin and Kelly 2012).
- Give up Fallibilism—vs. L2.
  Problem: We might believe p, although $Pr(p) < 1$.

Prominently held by (Levi 1980).

Revise the Generator

Ⓑ & Ⓟ & Ⓛ $\vdash$ $\neg$Ⓞ

Denying Structural Richness means to restrict possible *Pr*s.

Note that this does not imply that *Pr*s are restricted independently of the other constraints of rationality.

A very interesting case in point is The Stability Theory of Belief put forward by Hannes Leitgeb.

The Stability Theory of Belief

A stability condition:

$$\text{(S)}\quad \forall p, q:\ \ Bel(p),\ q \not\vdash \neg p \ \Rightarrow\ Pr(p|q) > 0.5$$

It can be shown:

$$\text{(B)}\ \&\ \text{(P)}\ \vdash\ \text{(L)}\ \text{iff}\ \text{(S)}$$

Here r in (L) equals $Pr(p)$, where p is the strongest proposition such that $Bel(p)$.

Solution: The stability condition restricts Pr depending on Bel.

(O) is invalidated since such believes are not stable.

Problem: Partition-dependence of possible rs $\Rightarrow$ "Contextualism"

A Non-Probabilistic Approach

Revise Degrees of Belief

Ⓑ & Ⓛ $\vdash$ ¬Ⓟ (& ¬Ⓞ)

(NB: Ⓑ, Ⓛ, and Ⓞ are already inconsistent.)

In general, things are quite open: A stability approach would serve probabilism.

Whether stability is the only way to uphold probabilism given Ⓑ and Ⓛ is an open question.

Revise Degrees of Belief

Given the more general approach:
$\forall Bel, Pr, r$: If (B) and (L) are satisfied for r, then ...

... one ends up with a fuzzy logic:

- $Pr(\neg p) = Pr(\top) - Pr(p)$
- $Pr(p\&q) = min(Pr(p), Pr(q))$
- $Pr(p \vee q) = max(Pr(p), Pr(q))$

Problem:
Does this enforce a complete different semantic/ontology for lottery cases:
Is $t_i = t_l$ not a true/false expression, but vague?
Are there other plausible interpretations, given the structure of fuzzy logic?

Summary

- The Lottery Paradox shows some incompatibility between rationality constraints for *Bel*, *Pr*, and Lockean bridging.
- Revision of the constraints for *Bel* leads to unattractive views like dialethism or impossibilities as shown by the review paradox.
- Revision of Lockean bridging is measure theoretically unorthodox.
- A restriction of *Pr* by stability constraints leads to (a weak form of) contextualism.
- Revision of *Pr* leads to fuzzyness whose interpretation for the lottery cases seems unintuitive.

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