Confirmation Based on Analogical Inference

Bayes meets Jeffrey

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Project Information

Publication(s):

- Feldbacher-Escamilla, Christian J. and Gebharter, Alexander (2020). "Confirmation Based on Analogical Inference. Bayes meets Jeffrey". In: *Canadian Journal of Philosophy* 50.2, pp. 174–194. DOI: 10.1017/can.2019.18.

Talk(s):

- Feldbacher-Escamilla, Christian J. and Gebharter, Alexander (2021-07-06/2021-07-06). *Confirmation Based on Analogical Inference. Bayes meets Jeffrey*. Research Seminar. Presentation (invited). Research Group: From Perception to Belief and Back Again. University of Bochum: University of Bochum.
- Feldbacher-Escamilla, Christian J. and Gebharter, Alexander (2019a-07-17/2019-07-19). *Confirmation Based on Analogical Inference. Bayes meets Jeffrey*. Conference. Presentation (contributed). The Annual Conference of the British Society for the Philosophy of Science (BSPS). University of Durham: BSPS.
- Feldbacher-Escamilla, Christian J. and Gebharter, Alexander (2018-10-26/2018-10-28). *Confirmation Based on Analogical Inference. Bayes meets Jeffrey*. Conference. Presentation (contributed). Analogical Reasoning in Science and Mathematics. LMU Munich: MCMP.
- Feldbacher-Escamilla, Christian J. and Gebharter, Alexander (2017-12-13/2017-12-15). *Analogical Reasoning in Medicine: Bayes meets Jeffrey*. Conference. Presentation (contributed). Issues in Medical Epistemology. University of Cologne: Department of Philosophy.

Project(s):

- DFG funded research unit *Inductive Metaphysics* (FOR 2495); subprojects: *Creative Abductive Inference and its Role for Inductive Metaphysics in Comparison to Other Metaphysical Methods* and *Statistical Causation Intervention and Freedom*.

Introduction

Sometimes evidence for a hypothesis cannot be directly observed.

This might be for ...

- ... theoretical reasons.
- ... practical reasons.
- ... moral reasons.

In such cases scientists often try to confirm a hypothesis H on the basis of analogical reasoning.

$\Rightarrow$ One uses evidence for a hypothesis H' about an analogous enough system in the process of confirming the problematic hypothesis H.

Problem: How exactly does the kind of confirmation involved here work?

Contents

Three Types of Analogical Inference

Characterisation

A general characterisation:

> *"An analogical argument is an explicit representation of analogical reasoning that cites accepted similarities between two systems [s, s'] in support of the conclusion that some further similarity exists" (Bartha 2010, p. 1 and p. 13)*

A general schema:

1. s' is similar to s in certain respects P_i and P'_i (where P_i and P'_i with $1 \leq i \leq n$ are similar).
2. s' has some additional feature Q'.
3. Therefore, s has feature Q (where Q and Q' are similar).

Characterisation

This is in line with representations as proposed by Hesse (1966, pp. 59f):

(source) properties of *sound* $[s']$	$[s]$ properties of *light* (target)
echoes $[P'_1(s')]$	$[P_1(s)]$ reflection
loudness $[P'_2(s')]$	$[P_2(s)]$ brightness
pitch $[P'_3(s')]$	$[P_3(s)]$ color
detected by ear $[P'_4(s')]$	$[P_4(s)]$ detected by eye
propagated in air $[Q'(s')]$	[Hence: $Q(s)$] propagated in ether

Hesse (1966, pp. 59f) also distinguished between:

- horizontal relations: similarity or identity
- vertical relations: causal dependence

Characterisation

Bartha (2010, p. 24) suggested to distinguish furthermore between:

- established relations (e.g. on empirical grounds)
- conjectured relations (e.g. merely considered to be possible)

Example: The similarity of s and s' w.r.t. P_i and P_i' (with $1 \leq i \leq 4$) was a de facto established relation, hence: horizontal relations simpliciter;

Similarity between Q and Q' was not established (also not excluded back then), hence: possible horizontal relation;

Bartha (2010) also applies this more subtle distinction to vertical relations: echoing and propagation in air are related via established causal relations, hence: vertical relation simpliciter.

Similarity between reflection and propagation in ether was not established, hence: possible vertical relation.

Characterisation

So, we can distinguish two types of analogical inferences:

1. inferring a (possible) horizontal relation
2. inferring a (possible) vertical relation

Here is the schema:

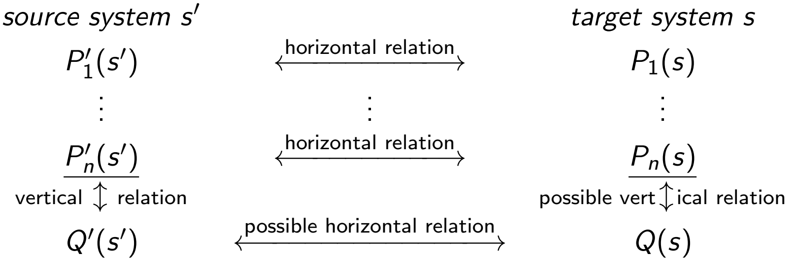


Figure: Two types of traditional analogical inferences

Characterisation

Example for an inference of type I: ether example

Example for an inference of type II: the violinist case of Thomson (1971)

Let us simplify the structure: H, E are the hypotheses and evidences of the target system; H', E' that of the source system.

Furthermore: Horizontal and vertical relations (possible and simpliciter) are confirmational relations.

Under this reading:

- Inferences of type I: confirmational impact of E' on E
- Inferences of type II: confirmational impact of E on H given that of E' on H'

Characterisation

Note: there is a third combinatorial possibility:

III inferring a (possible) diagonal relation: conf. impact of E' on H

(This is a not much discussed, but important case of *analogue simulation*.)

Overview of these relations:

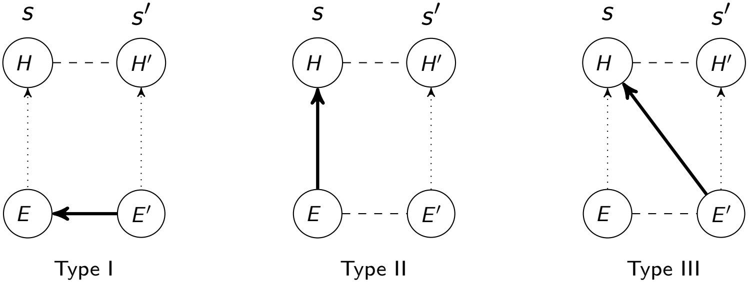


Figure: Three types of analogical inference in terms of confirmation

Characterisation

Modelling:

- Analogical inferences of type III are modelled by the Bayesian approach to confirmation of Dardashti, Thébault, and Winsberg (2015).
- Analogical inferences of type I can be easily embedded into their model.
- Analogical inferences of type II are modelled by our Jeffrey style expansion.

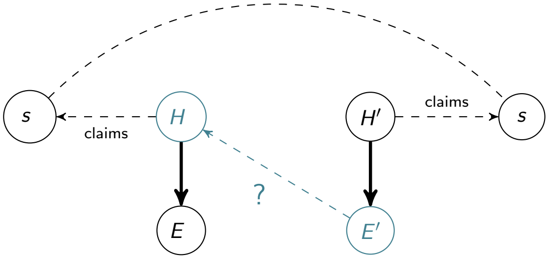
Note: analogical inferences type I and III are cases of *cross system confirmation*.

In the following: We present the model of Dardashti, Thébault, and Winsberg (2015) for type III inferences, briefly indicate how it also covers type I inferences, present some possible problems for *cross system confirmation*, and present our expansion for type II inferences.

Confirmation by Analogy Bayesian Style

Confirmation by Analogy Bayesian Style

A Rat Study Example



s ... immune system of humans

H ... hypothesis about s

E ... evidence for H

s' ... immune system of rats

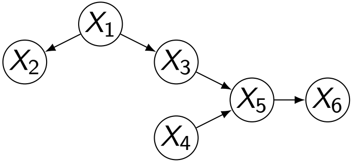
H' ... hypothesis about s'

E' ... evidence for H'

Bayesian Networks

A Bayesian network is a triple $\langle V, E, P\rangle$, such that ...

- V is a set of variables $X_1, \ldots, X_n$.
- E is a binary relation on V ($X_i \longrightarrow X_j$).
- P is a probability distribution over V.



$Par(X_i)$ ... the set of X_i's parents
$Des(X_i)$ ... the set of X_i's descendants

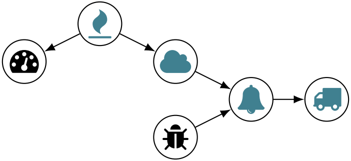
Bayesian Networks

Definition (Markov condition)

$\langle V, E, P\rangle$ satisfies the Markov condition iff every $X \in V$ is probabilistically independent of its non-descendants conditional on its parents (Pearl 2000, p.16).

Markov factorisation:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | Par(X_i))$$



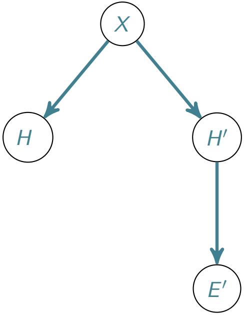
Dardashti, Thébault, and Winsberg (2015)'s Approach

The inference seems to crucially employ assumptions about the structure shared by the immune systems of humans and rats.

Dardashti, Thébault, and Winsberg (2015): Let's modell this shared structure by a variable X that is a common ancestor of H and H' in a Bayesian network.

$\Rightarrow$ E' indirectly confirms H Bayesian style:

$$P(H|E') > P(H)$$



Confirmation by Analogy Type III

Additional assumptions that have to hold for E' to indirectly confirm H:

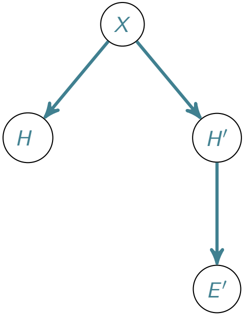
(i) $0 < P(X) < 1$

(ii) $P(H|X) > P(H|\neg X)$

(iii) $P(H'|X) > P(H'|\neg X)$

(iv) $P(E'|H') > P(E'|\neg H')$

$\Rightarrow P(H|E') > P(H)$
[Analogical Bayesian confirmation type III]



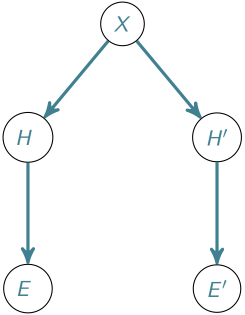
Confirmation by Analogy Type I

Additional assumptions that have to hold for E' to indirectly confirm E:

(i)–(iv) as before, plus:

(v) $P(E|H) > P(E|\neg H)$

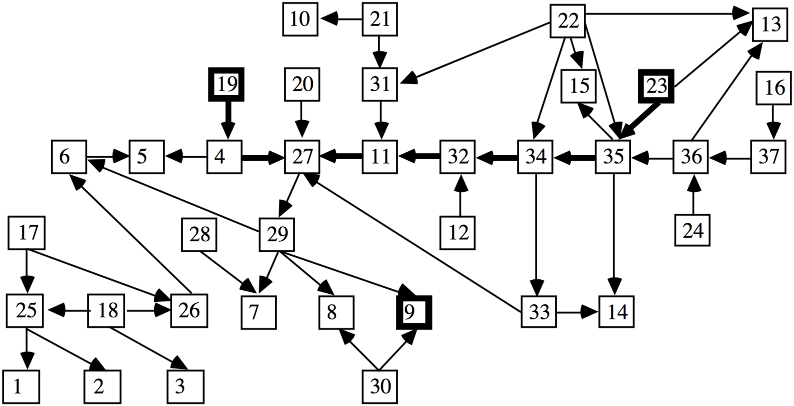
$\Rightarrow P(E|E') > P(E)$
[Bayesian confirmation via analogies type I]



Problems of Cross System Confirmation

1. Confirmation inflationism

ALARM belief network (reprinted and modified from (Spirtes, Glymour, and Scheines 2000, p.108))



19 . . . anaphylaxis occurs

23 . . . ventilation tube is kinked

9 . . . heart rate obtained from oximeter

Problems of Cross System Confirmation

2. Distrust in indirect evidence

At least sometimes scientists seem to be very cautious regarding confirmation (type III) by help of indirect evidence.

For example: It seems that experts as well as laymen prefer treatment that has been tested on humans over treatment that has only been tested on non-human model organisms, even if success rates for recovery were equally high.

Maybe the best explanation for this is that we do not accept results from studies on model organisms as evidence for hypotheses about humans.

Problems of Cross System Confirmation

3. Irrelevance of direct evidence

In analogical confirmation type III direct evidence E for the hypothesis H about the target system does not play any role for confirming H.

That E plays no role in confirming H stands in contrast to scientific practice.

It would be not clear, e.g., why scientists are looking for direct evidence E though they have already confirmed hypothesis H about the target system indirectly.

One can also find scenarios in which indirect evidence E' would have more confirmatory impact on H than direct evidence E has.

Confirmation by Analogy Jeffrey Style

Confirmation by Analogy Type II

Recall, for type II inferences we are looking for the confirmatory impact of E on H (target system), given that of E' on H' (source system).

The latter is: $Bconf(H'|E')$ which we might take as: $P(H'|E') - P(H')$

The former is (distance measure as proxy): $Bconf(H|E) = P(H|E) - P(H)$

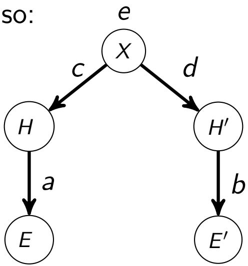
Now, we cannot simply make an inference of the form:

$$Bconf(H|E) = Bconf(H'|E')$$

Why? Because one cannot freely fix all relevant parameters.

Confirmation by Analogy Type II

Here is why exactly this is so:



$P(H|E)$ depends on parameters a, c, e and c is fixed by similarities.

However, $P(H'|E')$ depends similarly on parameters b, d, e.

Note, free is only a, hence we suggest for analogical inferences of type II: free parameter a is fixed by its pendant b, i.e.: $a = b$. Result:

If $c = d$ (perfect similarity), then $a = b \;\; \Rightarrow \;\; Bconf(H|E) = Bconf(H'|E')$

Confirmation by Analogy Type II: An Expansion

This type is not cross system confirmation, hence not prone to the problems.

However, it presupposes the E of the target system is available.

Still, one can also model type II confirmation with unknown E.

Here is how: We propose a 2-step approach to analogical confirmation.

Step 1. **Bayesian update** [analogical inference]:

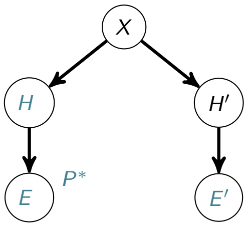
$$P^*(E) = P(E|E')$$

Step 2. **Jeffrey conditionalisation**:

$$P(H|E) \cdot P^*(E) + P(H|\neg E) \cdot P^*(\neg E)$$

Confirmation based on analogical inference:

$$P(H|E) \cdot P^*(E) + P(H|\neg E) \cdot P^*(\neg E) > P(H)$$



Confirmation by Analogy Type II: An Expansion

Recall Dardashti, Thébault, and Winsberg (2015)'s conditions that have to hold for E' to indirectly confirm H Bayesian style:

(i) $0 < P(X) < 1$

(ii) $P(H|X) > P(H|\neg X)$

(iii) $P(H'|X) > P(H'|\neg X)$

(iv) $P(E'|H') > P(E'|\neg H')$

Theorem

$P(H|E) \cdot P^*(E) + P(H|\neg E) \cdot P^*(\neg E) > P(H)$, *if (i)–(iv) and*

(v) $P(E|H) > P(E|\neg H)$

are satisfied.

Confirmation Measures

Measure for ordinary **Bayesian confirmation**:

$$\underbrace{Bconf(H|E')}_{type\ \ III} = P(H|E') - P(H)$$

B-J-confirmation measure based on our 2-step approach:

$$\underbrace{BJconf_{E'}(H|E)}_{type\ \ II} = \Big[P(H|E) \cdot P^*(E) + P(H|\neg E) \cdot P^*(\neg E)\Big] - P(H)$$

Theorem

$BJconf_{E'}(H|E) \leq Bconf(H|E')$, if (i)–(v) are satisfied.

Theorem

$BJconf_{E'}(H|E) \leq BJconf_{E}(H|E)$, if (i)–(v) are satisfied.

Summary

Problem: How exactly works confirmation based on analogical reasoning?

We identified 3 types of analogical inferences:

1. (cross system) impact of E' on E
2. impact of E on H based on that of E' on H'
3. (cross system) impact of E' on H

Dardashti, Thébault, and Winsberg (2015) model analogical inference of type III with a variable X representing the properties the target and the source system share.

This model can be straightforwardly expanded to cover type I inferences.

There are some possible problems for cross system confirmation: inflationism, distrust in indirect evidence, and irrelevance of direct evidence.

Our proposal for analogical inference of type II: Parameter mapping and a 2-step approach combining Bayesian update and Jeffrey conditionalisation.

A similar Bayesian model applies to abduction too (cf. our 2019-01).

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Appendix

Explicit form of the violinist example of Thomson (1971):

1. Generally, the violinist case s' is similar to the case of unwanted pregnancy s, i.e. P'_1, P_1 and Q', Q are similar.
2. A violinist's right to live $P'_1(s')$ does not establish a right to use someone else's body $Q'(s)$.
3. Therefore, also a baby's right to live $P_1(s)$ does not establish a right to use her mothers body $Q(s)$.

Appendix

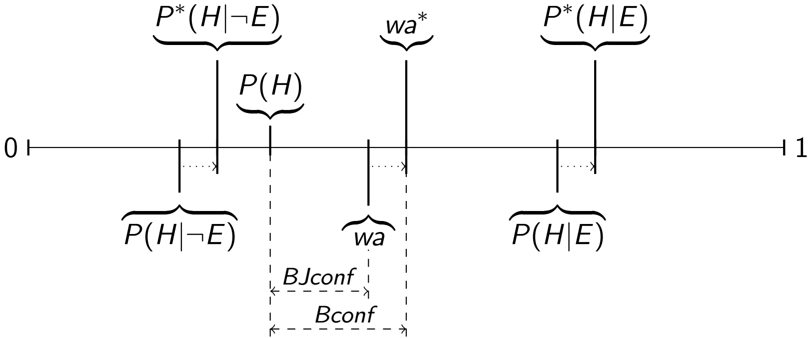

Figure: Diagramm for illustration of the confirmatory impact according to *BJconf* and *Bconf*