Diversity, Meta-Induction, and the Wisdom of the Crowd

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Summer 2013

Project Information

Publication(s):

- Feldbacher-Escamilla, Christian J. (2015). “Is the Equal-Weight View Really Supported by Positive Crowd Effects?” In: *Recent Developments in the Philosophy of Science: EPSA13 Helsinki*. Ed. by Mäki, Uskali et al. Heidelberg: Springer, pp. 87–98. DOI: 10.1007/978-3-319-23015-3_7.
- Feldbacher-Escamilla, Christian J. (2012). “Meta-Induction and the Wisdom of Crowds. A Comment”. In: *Analyse und Kritik* 34.2, pp. 367–382. DOI: 10.1515/auk-2012-0213.

Talk(s):

- Feldbacher-Escamilla, Christian J. (2014-06-19/2014-06-21). *Meta-Inductive Strategy Selection*. Conference. Presentation (contributed). International Conference of the Italian Society for Logic and Philosophy of Science (SILFS). University of Rome: SILFS.
- Feldbacher-Escamilla, Christian J. (2013a-08-28/2013-08-31). *Diversity, Meta-Induction, and the Wisdom of the Crowd*. Conference. Presentation (contributed). EPSA13. Conference of the European Philosophy of Science Association (EPSA). University of Helsinki: EPSA.
- Feldbacher-Escamilla, Christian J. (2013b-07-04/2013-07-05). *Diversity, Meta-Induction, and the Wisdom of the Crowd*. Conference. Presentation (contributed). The Annual Conference of the British Society for the Philosophy of Science (BSPS). University of Exeter: BSPS.

Project(s):

- DFG: *New Frameworks of Rationality* (SPP1516); *The Role of MI in Human Reasoning*.

Introduction

Two core problems of philosophy of science and epistemology are the questions:

1. How to justify scientific methods, especially inductive methods?
2. How to deal with disagreements among experts (peers)?

There are two approaches that try to cope with these problems by putting them on a meta-level:

1. Induction $\Rightarrow$ Meta-induction
2. Peer disagreement $\Rightarrow$ Wisdom of the crowd

In this talk we will consider both approaches and problems arising within an overall approach.

Contents

1. Meta-Induction
   - Object-Level
   - Meta-Level
2. The Wisdom of the Crowd
   - Object-Level
   - Meta-Level
3. The Two Strategies Getting Together

Meta-Induction

The Problem of Induction

E.g.: How to justify enumerative induction?

1. $P(c_1)$
2. $\vdots$
3. $P(c_n)$
4. Hence: $P(c_{n+1})$

There is of course no deductive justification: $P(c_1)\& \ldots \& P(c_n) \not\vdash P(c_{n+1})$.

And there is probably also no inductive justification: It doesn't hold generally that $conf(P(c_{n+1})|P(c_1)\& \ldots \& P(c_n)) > conf(P(c_{n+1})|\top)$

Putting the Problem on a Meta-Level ...

One way to give a justification for at least some inductive methods is to bring them onto a meta-level.

The idea is not to justify induction *per se*, but induction *per comparison* with its competitors.

This idea traces back to Hans Reichenbach's so-called '*best alternative approach*'.

Putting the Problem on a Meta-Level ...

Let's see some more details (Schurz 2008), (Schurz 2009)!

Assume $\alpha_1, \ldots, \alpha_n$ to be all agents within a setting and $\alpha_{_{wMI}}$, a meta-inductivistic agent, to be one of them.

Assume V_{α_T} to be the truth (α_T is a truth-teller).

Then we can define the prediction success of an agent α_i by first measuring its errors:

$$E_{\alpha_i}(x) = (V_{\alpha_{_T}}(x) - V_{\alpha_i}(x))^2 \quad (1)$$

And then summing them up:

$$succ_{x,t}(\alpha_i) = \frac{\sum_{i=1}^{t} 1 - \frac{E_{\alpha_i}(x+i)}{V_{max}{}^2}}{t} \quad (2)$$

Putting the Problem on a Meta-Level ...

With the help of the success rate one can define an attractivity measure which represents the attractivity of an agent for the meta-inductivist α_{wMI} according to the success rate:

$$attr_{x,t}(\alpha_i) = max(\{0, succ_{x,t}(\alpha_i) - succ_{x,t}(\alpha_{wMI})\}) \tag{3}$$

And with the help of the attractivity measure one can define weighting coefficients for the meta-inductivist α_{wMI} which relativize the attractivity of an agent to the whole group:

$$c_\alpha(x, t) = \frac{attr_{x,t}(\alpha)}{\sum_{i=1}^{n} attr_{x,t}(\alpha_i)} \tag{4}$$

Finally, with those weighting coefficients a meta-inductivist can construct a strategy in a very easy way:

$$V_{\alpha_{wMI}}(x) = c_{\alpha_1}(x, t) \cdot V_{\alpha_1}(x) + \cdots + c_{\alpha_n}(x, t) \cdot V_{\alpha_n}(x) \tag{5}$$

An Example

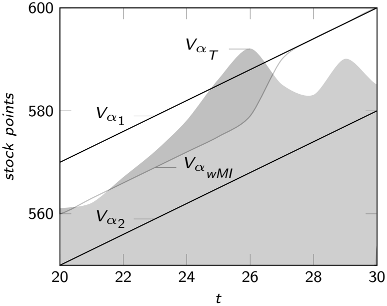

Figure: Setting: $\{\alpha_1, \alpha_2, \alpha_{wMI}\}$

. . . And Getting a Partial Solution

Optimality Constraint:

α is optimal in its predictions if its success-rate is maximal in the long run.

Theorem (cf. Thorn and Schurz 2012, Theorem 2):

α_{wMI}'s success-rate is maximal in the long run (if the individual error functions E_{α_i} are convex $\forall i \leq n$).

The Wisdom of the Crowd

The Problem of Peer Disagreement

What if two epistemic peers (same training, same data) disagree about an event's outcome.

One answer to the problem of peer disagreement is given by the equal weight view (cf. Elga 2007):

$V_{\alpha_i}(x) = \frac{V_{\alpha_1}(x)+\cdots+V_{\alpha_n}(x)}{n}$ I.e.: $c_{\alpha_i}(x, t) = \frac{1}{n}$

And one argument in favour of the equal weight view can be constructed by: putting the problem on a meta-level.

Putting the Problem on a Meta-Level ...

If one performs an equal weight view, then one can make use of a wisdom of the crowd effect:
Recall the measure for the error of an individual's prediction (cf. Krogh and Vedelsby 1995, p.232):

$$E_{\alpha_i}(x) = (V_{\alpha_T}(x) - V_{\alpha_i}(x))^2$$

Now, let the group's prediction be the average of the individual's prediction (cf. Krogh and Vedelsby 1995, p.232):

$$V_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) = \frac{\sum_{i=1}^{n} V_{\alpha_i}(x)}{n} \tag{6}$$

So, the group's prediction error can be calculated like a single agent's prediction error (cf. Krogh and Vedelsby 1995, p.232):

$$E_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) = (V_{\alpha_T}(x) - V_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x))^2 \tag{7}$$

Putting the Problem on a Meta-Level ...

If one compares the group's prediction error with the individuals' error, as measured via (cf. Krogh and Vedelsby 1995, p.232):

$$E_{\varnothing\{\alpha_1,\ldots,\alpha_n\}}(x) = \frac{\sum_{i=1}^{n} E_{\alpha_i}(x)}{n} \tag{8}$$

Then one can observe:

$$E_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) \leq E_{\varnothing\{\alpha_1,\ldots,\alpha_n\}}(x) \tag{9}$$

So, the crowd beats the average individual.

Furthermore, one can show that the crowd performs the better, the more diverse it is:

Putting the Problem on a Meta-Level . . .

Let the prediction diversity of an individual be measured by its deviation from the average prediction:

$$D_{\alpha_i}(x) = (V_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) - V_{\alpha_i}(x))^2 \tag{10}$$

So, the more an individual's prediction converges to the average prediction, the less diverse it is.

Now, let's measure the diversity within a group just by averaging the individual's diversity:

$$D_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) = \frac{\sum\limits_{i=1}^{n} D_{\alpha_i}(x)}{n} \tag{11}$$

Putting the Problem on a Meta-Level . . .

Then one can observe (cf. Krogh and Vedelsby 1995, p.232):

$$E_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) = E_{\varnothing\{\alpha_1,\ldots,\alpha_n\}}(x) - D_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) \tag{12}$$

So, one can say that, in general, it holds that the lower the average error or the higher the diversity within a group, the lower the error of the group's prediction.

Note again that the method for building up the group's prediction is a meta (but no inductive) method.

… And Getting a Partial Solution

By putting the argumentation to a meta-level, one gets a partial solution to the problem of peer disagreement:

If one performs an equal weighting method, then one can make use of a wisdom of the crowd effect.

And by this one performs at least as good as an average individual.

Note: Simulations show that sometimes equal weighting is optimal and sometimes is not (cf. Douven 2010).

The Two Strategies Getting Together

Trying to Solve Both Problems at Once

So, on the one hand we have a partial solution to the problem of induction by optimality results on meta-induction.

And on the other hand we have a partial solution to the problem of peer disagreement by wisdom of the crowd effects in equal weighting.

What, if one tries to solve both problems at once?

I.e.: What, if one wants to perform equal weighting while also using meta-induction (cf. Thorn and Schurz 2012)?

Trying to Solve Both Problems at Once

The influence is twofold – recall:

$$E_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x) = E_{\varnothing\{\alpha_1,\ldots,\alpha_n\}}(x) - D_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x)$$

1. Since meta-induction is an optimal method in the long run, performing it decreases the error on average: $E_{\varnothing\{\alpha_1,\ldots,\alpha_n\}}(x)$
2. But since meta-induction is a meta-method, performing it also decreases the diversity within a group: $D_{\Gamma=\{\alpha_1,\ldots,\alpha_n\}}(x)$

Principally there are two cases that have to be considered (cf. Thorn and Schurz 2012):

1. A meta-inductivist enters/leaves a prediction game
2. A meta-inductivist replaces another agent in a prediction game

(The investigation can be subordinated to institutional design.)

Institutional Design: Meta-Inductivistic Invasion

If a meta-inductivist operates optimally in a situation, then adding it to a group doesn't harm the wisdom of the crowd effect (cf. Thorn and Schurz 2012, p.346):

> *Let* Γ_1 *be a group of agents* $\alpha_1, \ldots, \alpha_n$ *and* $\Gamma_2 = \Gamma_1 \cup \{\alpha_{wMI}\}$. *Then it holds that* $E_{\Gamma_2}(x) \leq E_{\Gamma_1}(x)$ *provided that* $E_{\alpha_{wMI}}(x) \leq E_{\alpha_i}(x)$ $\forall i \leq n$.

But of course, a meta-inductivist operates very often sub-optimally:

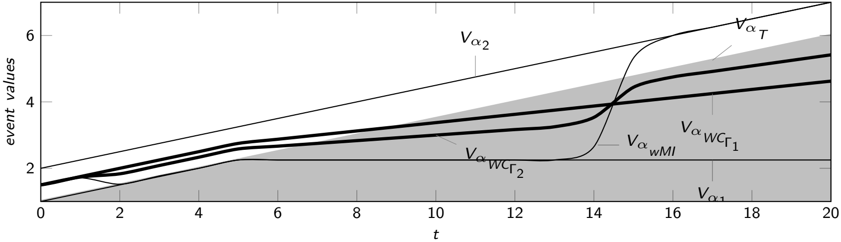

Figure: Setting: $\Gamma_1 = \{\alpha_1, \alpha_2\}$, $\Gamma_2 = \{\alpha_1, \alpha_2, \alpha_{wMI}\}$;

Institutional Design: Meta-Inductivistic Invasion

Nevertheless there holds a positive result for the long run:

In *summing up* the wisdom of the crowd effects in a series of predictions *adding* a meta-inductivist *doesn't harm* the summed up wisdom of the crowd effect in the long run.

Institutional Design: Meta-Inductivistic Replacement

A meta-inductivist with a . . .

By replacing predictors with meta-inductivistic agents summing up the wisdom of the crowd effects is of no use.

Paul D. Thorn and Gerhard Schurz think nevertheless that even in meta-inductivistic replacement there is a positive tendency with respect to the wisdom of the crowd:

> *"we would also like to suggest, in contradiction to Page (2007, 208), that collective diversity [. . .] is not as important as individual ability [. . .] to the wisdom of a crowd [. . . .]." (cf. Thorn and Schurz 2012, p.345)*

Considering the equation expressing the wisdom of the crowd effect, there is no reason to distinguish an influence between average error and diversity.

Institutional Design: Meta-Inductivistic Replacement

And also practically seen, the diversity factor is very important:

- Interdisciplinary research (diversity at the cost of competence)
- Positive discrimination (diversity at no cost of competence)
- Scientific pluralism (diversity at the cost of competence measurement)

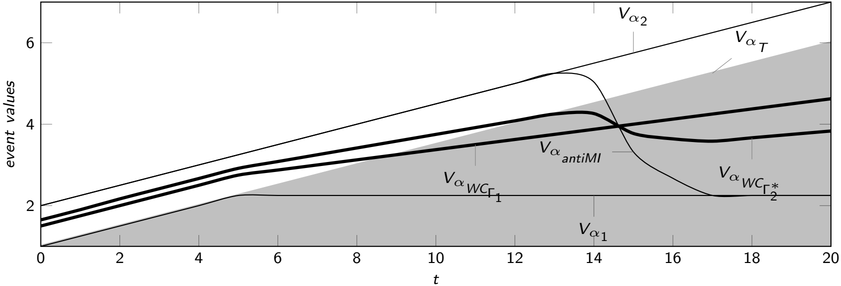

Figure: Setting: $\Gamma_1 = \{\alpha_1, \alpha_2\}$, $\Gamma_2^* = \{\alpha_1, \alpha_2, \alpha_{antiMI}\}$; by considering the distances of $V_{\alpha_{WC_{\Gamma_1}}}$ and $V_{\alpha_{WC_{\Gamma_2^*}}}$ to the truth (V_{α_T}), adding the meta-anti-inductivist α_{antiMI} to a group exceptionally increases (here: in the frame $10 < t < 15$) the wisdom of the crowd effect.

Institutional Design: Meta-Inductivistic Replacement

According to, e.g., Paul Feyerabend some perhaps seldom, but nevertheless very *important parts of the history of science* are estimated as being *successful due to diversity* or plurality at the cost of competence with respect to an old paradigm (cf. Feyerabend 1993).

So, in general it holds that one can provide partial solutions to both problems

- the problem of induction and
- the problem of peer disagreement

at once by adding meta-inductivistic methods to a group.

But one cannot generally improve a group's performance by institutional design in the sense of meta-inductivistic replacement.

Summary

- Problem of induction ⇒ Meta-induction
- Problem of peer disagreement ⇒ Wisdom of the crowd
- Meta-induction may undermine the wisdom of the crowd effect
- But in the long run adding meta-inductive methods increase the summed up wisdom of the crowd effects

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